Assigned Work

The following identities involve the reciprocal, quotient, and Pythagorean relationships. Prove each one.

1. $\sin x \tan x = \sec x - \cos x$
2. $\cos^4 x - \sin^4 x = 1 - 2\sin^2 x$
3. $\csc^2 x + \sec^2 x = \csc^2 x \sec^2 x$
4. $\cos^2 x \cos^2 y + \sin^2 x \sin^2 y + \sin^2 x \cos^2 y + \sin^2 y \cos^2 x = 1$
5. $\sec^2 x - \sec^2 y = \tan^2 x - \tan^2 y$
6. $\dfrac{\tan x + \tan y}{\cot x + \cot y} = (\tan x)(\tan y)$
7. $(\sec x - \cos x)(\csc x - \sin x) = \dfrac{\tan x}{1 + \tan^2 x}$
8. $\cos^6 x + \sin^6 x = 1 - 3\sin^2 x + 3\sin^4 x$
9. $\sec^6 x - \tan^6 x = 1 + 3\tan^2 x \sec^2 x$

The following involve the addition and subtraction formulas.

10. $1 + \cot x \tan y = \dfrac{\sin(x + y)}{\sin x \cos y}$
11. $\cos(x + y)\cos y + \sin(x + y)\sin y = \cos x$
12. $\sin x - \tan y \cos x = \dfrac{\sin(x - y)}{\cos y}$
13. $\cos\left(\dfrac{3\pi}{4} + x\right) + \sin\left(\dfrac{3\pi}{4} - x\right) = 0$
14. $\dfrac{\tan\left(\dfrac{\pi}{4} + x\right) - \tan\left(\dfrac{\pi}{4} - x\right)}{\tan\left(\dfrac{\pi}{4} + x\right) + \tan\left(\dfrac{\pi}{4} - x\right)} = 2\sin x \cos x$
15. $\sin(x + y)\sin(x - y) = \cos^2 y - \cos^2 x$
16. $\tan(x + y)\tan(x - y) = \dfrac{\sin^2 x - \sin^2 y}{\cos^2 x - \sin^2 y}$
17. $\dfrac{\tan(x - y) + \tan y}{1 - \tan(x - y)\tan y} = \tan x$
18. $\sin 5x = \sin x(\cos^2 2x - \sin^2 2x) + 2\cos x \cos 2x \sin 2x$

The following involve related and co-related angles.

19. $\sin\left(\dfrac{\pi}{2} - x\right)\cot\left(\dfrac{\pi}{2} + x\right) = -\sin x$
20. $\cos(-x) + \cos(\pi - x) = \cos(\pi + x) + \cos x$

21. $\dfrac{\sin(\pi - x)}{\tan(\pi + x)} \dfrac{\cot\left(\dfrac{\pi}{2} - x\right)}{\tan\left(\dfrac{\pi}{2} + x\right)} \dfrac{\cos(2\pi - x)}{\sin(-x)} = \sin x$

22. $\dfrac{\sin(-x)}{\sin(\pi + x)} - \dfrac{\tan\left(\dfrac{\pi}{2} + x\right)}{\cot x} + \dfrac{\cos x}{\sin\left(\dfrac{\pi}{2} + x\right)} = 3$

23. $\dfrac{\csc(\pi - x)}{\sec(\pi + x)} \dfrac{\cos(-x)}{\cos\left(\dfrac{\pi}{2} + x\right)} = \cot^2 x$

24. $\dfrac{\cos\left(\dfrac{\pi}{2} + x\right)\sec(-x)\tan(\pi - x)}{\sec(2\pi + x)\sin(\pi + x)\cot\left(\dfrac{\pi}{2} - x\right)} = -1$

25. $\dfrac{\sin(\pi - x)\cos(\pi + x)\tan(2\pi - x)}{\sec\left(\dfrac{\pi}{2} + x\right)\csc\left(\dfrac{3\pi}{2} - x\right)\cot\left(\dfrac{3\pi}{2} + x\right)} = \sin^4 x - \sin^2 x$

The following involve the double angle formulas.

26. $\dfrac{\sin 2x}{1 + \cos 2x} = \tan x$

27. $\dfrac{1 + \cos x}{\sin x} = \cot \dfrac{x}{2}$

28. $2 \csc 2x = \sec x \csc x$

29. $2 \cot 2x = \cot x - \tan x$

30. $\dfrac{\cos 2x}{1 + \sin 2x} = \tan\left(\dfrac{\pi}{4} - x\right)$

31. $\dfrac{\cos x - \sin x}{\cos x + \sin x} = \sec 2x - \tan 2x$

32. $\dfrac{1 - \cos 2x + \sin 2x}{1 + \cos 2x + \sin 2x} = \tan x$

33. $\cos^6 x - \sin^6 x = \cos 2x\left(1 - \frac{1}{4}\sin^2 2x\right)$

34. $4(\cos^6 x + \sin^6 x) = 1 + 3\cos^2 2x$

35. $\sec x - \tan x = \tan\left(\dfrac{\pi}{4} - \dfrac{x}{2}\right)$

36. $\dfrac{\sin 2x}{1 + \cos 2x} \dfrac{\cos x}{1 + \cos x} = \tan \dfrac{x}{2}$

The following involve a variety of formulas and identities.

37. $\sin^2 x + \cos^4 x = \cos^2 x + \sin^4 x$

38. $\tan x - \cot x = (\tan x - 1)(\cot x + 1)$

39. $\cos x = \sin x \tan^2 x \cot^3 x$

40. $(\sin x + \cos x)(\tan x + \cot x) = \sec x + \csc x$

41. $\sin^4 x + \cos^4 x = \sin^2 x(\csc^2 x - 2\cos^2 x)$

42. $\sin^3 x + \cos^3 x = (1 - \sin x \cos x)(\sin x + \cos x)$

43. $\cos\left(\frac{\pi}{12} - x\right)\sec\frac{\pi}{12} - \sin\left(\frac{\pi}{12} - x\right)\csc\frac{\pi}{12} = 4\sin x$

44. $\tan(x - y) + \tan(y - z) = \dfrac{\sec^2 y\,(\tan x - \tan z)}{(1 + \tan x \tan y)(1 + \tan y \tan z)}$

45. $\sin 8x = 8 \sin x \cos x \cos 2x \cos 4x$

46. $\sin x = 1 - 2\sin^2\left(\frac{\pi}{4} - \frac{x}{2}\right)$

47. $\sin(x + y) + \sin(x - y) = 2\sin x \cos y$

48. $\dfrac{\sin(x - y)}{\sin x \sin y} + \dfrac{\sin(y - z)}{\sin y \sin z} + \dfrac{\sin(z - x)}{\sin z \sin x} = 0$

49. $\tan x + \tan(\pi - x) + \cot\left(\frac{\pi}{2} + x\right) = \tan(2\pi - x)$

50. $\sin\left(\frac{\pi}{2} + x\right)\cos(\pi - x)\cot\left(\frac{3\pi}{2} + x\right)$
$= \sin\left(\frac{\pi}{2} - x\right)\sin\left(\frac{3\pi}{2} - x\right)\cot\left(\frac{\pi}{2} + x\right)$

51. $\tan\left(\frac{\pi}{2} - x\right) - \cot\left(\frac{3\pi}{2} - x\right) + \tan(2\pi - x) - \cot(\pi - x)$
$= \dfrac{4 - 2\sec^2 x}{\tan x}$

52. $\tan(x + y + z) = \dfrac{\tan x + \tan y + \tan z - \tan x \tan y \tan z}{1 - \tan x \tan y - \tan x \tan z - \tan y \tan z}$

53. $\csc^2\left(\frac{\pi}{2} - x\right) = 1 + \sin^2 x \csc^2\left(\frac{\pi}{2} - x\right)$

54. $\tan\left(\frac{\pi}{4} + x\right) + \tan\left(\frac{\pi}{4} - x\right) = 2\sec 2x$

55. $\dfrac{1 - \sin 2x}{\cos 2x} = \dfrac{\cos 2x}{1 + \sin 2x}$

56. $\dfrac{\sin 4x}{1 - \cos 4x} \times \dfrac{1 - \cos 2x}{\cos 2x} = \tan x$

LS	RS
1 $\sin x \tan x$	$\sec x - \cos x$
$= \sin(x) \dfrac{\sin x}{\cos x}$	$= \dfrac{1}{\cos x} - \cos x$
$= \dfrac{\sin^2 x}{\cos x}$	$= \dfrac{1 - \cos^2 x}{\cos x}$
	$= \dfrac{\sin^2 x}{\cos x}.$

$\therefore LS = RS$

LS $\qquad\qquad\qquad\qquad$ RS

2 $\cos^4 x - \sin^4 x \qquad\qquad 1 - 2\sin^2 x$

$= (\cos^2 x - \sin^2 x)(\cos^2 x + \sin^2 x)$

$= (\cos^2 x - \sin^2(x))(1)$

$= 1 - \sin^2 x - \sin^2 x$

$= 1 - 2\sin^2 x.$

$\therefore$ LS = RS

LS

RS

3 $\csc^2 x + \sec^2 x$

$\csc^2 x \sec^2 x$

$= \frac{1}{\sin^2 x} + \frac{1}{\cos^2 x}$

$= \frac{1}{\sin^2 x} \cdot \frac{1}{\cos^2 x}$

$= \frac{\sin^2 x}{\sin^2 x \cos^2 x} + \frac{\cos^2 x}{\sin^2 x \cos^2 x}$

$= \frac{1}{\sin^2 x \cos^2 x}$

$= \frac{1}{\sin^2 x \cos^2 x}$

$\therefore$ LS = RS

LS $\qquad\qquad\qquad$ RS

4 $\underbrace{\cos^2 x \cos^2 y + \sin^2 x \sin^2 y + \sin^2 x \cos^2 y + \sin^2 y \cos^2 x}$

$= (\cos^2 x + \sin^2 x)\cos^2 y + \sin^2 y(\sin^2 x + \cos^2 x)$

$= 1 \cos^2 y + \sin^2 y (1.)$

$= 1.$

RS.

1

$\therefore LS = RS$

LS | RS

5. $\sec^2 x - \sec^2 y$ | $\tan^2 x - \tan^2 y$

$\dfrac{1}{\cos^2 x} - \dfrac{1}{\cos^2 y}$ | $= \dfrac{\sin^2 x}{\cos^2 x} - \dfrac{\sin^2 y}{\cos^2 y}$

$\dfrac{\cos^2 y - \cos^2 x}{\cos^2 x \cos^2 y}$ | $= \dfrac{\sin^2 x - \sin^2 y}{\cos^2 x \cos^2 y}$

$= \dfrac{(1-\cos^2 x) - (1-\cos^2 y)}{\cos^2 x \cos^2 y}$

$= \dfrac{\cos^2 y - \cos^2 x}{\cos^2 x (\cos^2 y)}$

$\therefore$ LS = RS

LS $\qquad\qquad\qquad\qquad$ R.S.

6 $\dfrac{\tan x + \tan y}{\cot x + \cot y}$ $\qquad\qquad$ $\tan x \tan y$.

$$= \frac{\tan x + \tan y}{\left(\frac{1}{\tan x} + \frac{1}{\tan y}\right)}$$

$$= \left(\tan x + \tan y\right) \div \left(\frac{\tan y + \tan x}{\tan y \tan x}\right).$$

$$= \cancel{(\tan x + \tan y)} \times \frac{\tan y \tan x}{\cancel{(\tan y + \tan x)}}$$

$$= \tan y \tan x$$

LS

7. $(\sec x - \cos x)(\csc x - \sin x)$

RS

$\frac{\tan x}{1+\tan^2 x}$

$= \left(\frac{1}{\cos x} - \cos x\right)\left(\frac{1}{\sin x} - \sin x\right)$

$= \frac{1}{\sin x \cos x} - \frac{\cos x}{\sin x} - \frac{\sin x}{\cos x} + \cos x \sin x$

$= \frac{1 - \cos^2 x - \sin^2 x + \cos^2 x \sin^2 x}{\sin x \cos x}$

$= \frac{1 - (1) + \cos^2 x \sin^2 x}{\sin x \cos x}$

$= \cos x \sin x$

RS

$\frac{\tan x}{1+\tan^2 x}$

$\frac{\sin x}{\cos x} \div \left(1 + \left(\frac{\sin x}{\cos x}\right)^2\right)$

$\frac{\sin x}{\cos x} \div \left(\frac{\cos^2 x + \sin^2 x}{\cos^2 x}\right)$

$\frac{\sin x}{\cos x} \times \frac{\cos^2 x}{1} = \sin x \cos$

$\therefore LS = RS$

LS $\qquad\qquad\qquad\qquad$ RS

8 $\cos^6 x + \sin^6 x \qquad\qquad 1 - 3\sin^2 x + 3\sin^4 x$

$= (\cos^2 x + \sin^2 x)(\cos^4 x - \cos^2 x \sin^2 x + \sin^4 x)$

$= 1 \quad ((1-\sin^2 x)^2 - (1-\sin^2 x)\sin^2 x + \sin^4 x)$

$= ((1 - 2\sin^2 x + \sin^4 x) - (\sin^2 x - \sin^4 x) + \sin^4 x)$

$= 1 - 3\sin^2 x + 3\sin^4 x$

$\therefore$ LS = RS.

9.

LS	RS
$\sec^6 x - \tan^6 x$	$1 + 3\tan^2 x \sec^2 x$
$(1+\tan^2 x)^3 - \tan^6 x$	$1 + 3\tan^2 x(1+\tan^2 x)$
$= (1+\tan^2 x)(1+2\tan^2 x+\tan^4 x) - \tan^6 x$	$1 + 3\tan^2 x + 3\tan^4 x$

$= 1 + 2\tan^2 x + \tan^4 x + \tan^2 x + 2\tan^4 x + \cancel{\tan^6 x} - \cancel{\tan^6 x}$

$= 1 + 3\tan^2 x + 3\tan^4 x.$

10. $1 + \cot x \tan y = \dfrac{\sin(x+y)}{\sin x \cos y}$

$LS\ \cancel{RS} = 1 + \cot x \tan y$ $\quad RS = \dfrac{\sin x \cos y + \cos x \sin y}{\sin x \cos y}$

$$= \frac{\sin x \cos y}{\sin x \cos y} + \frac{\cos x \sin y}{\sin x \cos y}$$

$$= 1 + \frac{\cos x}{\sin x} \cdot \frac{\sin y}{\cos y}$$

$$= 1 + \cot x \tan y$$

$LS = RS$

11. $\cos(x+y)\cos y + \sin(x+y)\sin y = \cos x$

$LS = (\cos x\cos y - \sin x\sin y)\cos y + (\sin x\cos y + \cos x\sin y)\sin y$ $RS = \cos x$

$= \cos x\cos^2 y - \cancel{\sin x\sin y\cos y} + \cancel{\sin x\sin y\cos y} + \cos x\sin^2 y$

$= \cos x(\cos^2 y + \sin^2 \cancel{x}y)$

$= \cos x \quad (1)$

$= \cos x$

$LR = RS.$

12. $\sin x - \tan y \cos x = \dfrac{\sin(x-y)}{\cos y}$

$LS = \sin x - \tan y \cos x$ $\quad$ $RS = \dfrac{\sin(x-y)}{\cos y}$

$$= \frac{\sin x \cos y - \cos x \sin y}{\cos y}$$

$$= \frac{\sin x \cancel{\cos y}}{\cancel{\cos y}} - \frac{\cos x \sin y}{\cos y}$$

$$= \sin x - \cos x \tan y$$

$LS = RS$

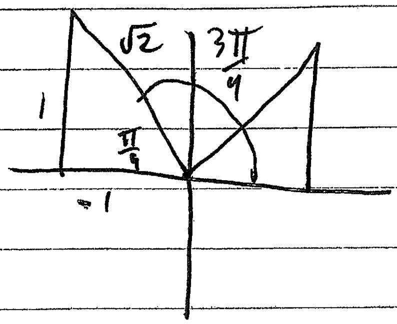
LS RS

13 $\cos\left(\frac{3\pi}{4}+x\right)+\sin\left(\frac{3\pi}{4}-x\right)$ | 0

$= \cos\frac{3\pi}{4}\cos x - \sin\frac{3\pi}{4}\sin x + \sin\frac{3\pi}{4}\cos x - \cos\frac{3\pi}{4}\sin x$

$= -\frac{1}{\sqrt{2}}\cos x - \frac{1}{\sqrt{2}}\sin x + \frac{1}{\sqrt{2}}\cos x - \left(-\frac{1}{\sqrt{2}}\right)\sin x$

$= 0$



LS RS

14. $\dfrac{\tan(\frac{\pi}{4}+x)-\tan(\frac{\pi}{4}-x)}{\tan(\frac{\pi}{4}+x)+\tan(\frac{\pi}{4}-x)}$ $\qquad$ $2\sin x\cos x$.

$$=\frac{\dfrac{\tan\frac{\pi}{4}+\tan x}{1-\tan\frac{\pi}{4}\tan x}-\dfrac{\tan\frac{\pi}{4}-\tan x}{1+\tan\frac{\pi}{4}\tan x}}{\dfrac{\tan\frac{\pi}{4}+\tan x}{1-\tan\frac{\pi}{4}\tan x}+\dfrac{\tan\frac{\pi}{4}-\tan x}{1+\tan\frac{\pi}{4}\tan x}}$$

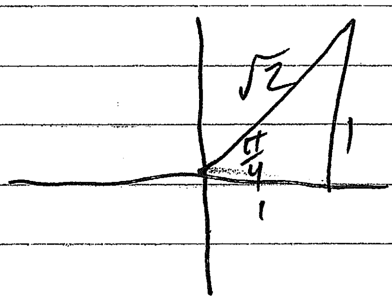
$$=\left(\frac{1+\tan x}{1-1(\tan x)}-\frac{1-\tan x}{1+1(\tan x)}\right)\div\left(\frac{1+\tan x}{1-1(\tan x)}+\frac{1-\tan x}{1+(1)\tan x}\right)$$

$$=\frac{(1+\tan x)(1+\tan x)-(1-\tan x)(1-\tan x)}{(1-\tan x)(1+\tan x)}\div\frac{(1+\tan x)(1+\tan x)+(1-\tan x)(1-\tan}{(1-\tan x)(1+\tan x)}$$

$$=\frac{(1+2\tan x+\tan^2x)-(1-2\tan x+\tan^2x)}{(1-\cancel{\tan x})(1+\cancel{\tan x})}\times\frac{(1-\cancel{\tan x})(1+\cancel{\tan x})}{(1+2\tan x+\tan^2x)+(1-2\tan x+\tan}$$

$$=\frac{4\tan x}{2+2\tan^2x}$$

$$=\frac{2\tan x}{1+\tan^2x}$$



$\tan\frac{\pi}{4}=1$

$$=\frac{2\tan x}{\sec^2x}$$

$$=\frac{2\sin x}{\cos x}\div\frac{1}{\cos^2x}=\frac{2\sin x\cos^2x}{\cos x}\quad\therefore LS=RS$$

15 $LS = \sin(x+y)\sin(x-y)$ $\qquad$ $RS = \cos^2 y - \cos^2 x$

$= (\sin x \cos y + \cos x \sin y)(\sin x \cos y - \cos x \sin y)$

notice difference of squares $(a+b)(a-b) = a^2 - b^2$.

$= (\sin^2 x \cos^2 y - \cos^2 x \sin^2 y)$

$= \left((1-\cos^2 x)\cos^2 y - \cos^2 x (1-\cos^2 y) \right)$

$= \cos^2 y - \cancel{\cos^2 y \cos^2 x} - \cos^2 x + \cancel{\cos^2 x \cos^2 y}$

$= \cos^2 y - \cos^2 x$

$LS = RS.$

16 $LS = \tan(x+y)\tan(x-y)$ $\qquad RS = \dfrac{\sin^2 x - \sin^2 y}{\cos^2 x - \sin^2 y}$

$$= \frac{\sin(x+y)}{\cos(x+y)} \cdot \frac{\sin(x-y)}{\cos(x-y)}$$

$$= \frac{(\sin x\cos y + \cos x\sin y)}{(\cos x\cos y - \sin x\sin y)} \cdot \frac{(\sin x\cos y - \cos x\sin y)}{(\cos x\cos y + \sin x\sin y)}$$

$$= \frac{\sin^2 x\cos y^2 - \cos^2 x\sin^2 y}{\cos^2 x\cos^2 y - \sin^2 x\sin^2 y}$$

$$= \frac{\sin^2 x(1-\sin^2 y) - (1-\sin^2 x)(\sin^2 y)}{\cos^2 x(1-\sin^2 y) - (1-\cos^2 x)(\sin^2 y)}$$

$$= \frac{\sin^2 x - \sin^2 x\sin^2 y - \sin^2 y + \sin^2 x\sin^2 y}{\cos^2 x - \cos^2 x\sin^2 y - \sin^2 y + \cos^2 x\sin^2 y}$$

$$= \frac{\sin^2 x - \sin^2 y}{\cos^2 x - \sin^2 y}$$

$LS = RS$

LS RS

17 $\dfrac{\tan(x-y) + \tan y}{1 - \tan(x-y)\tan y}$ $\tan x$

$= \tan((x-y)+y)$

$= \tan x$

	LS	RS
18	$\sin 5x$	$\sin x(\cos^2 2x - \sin^2 2x) + 2\cos x \cos 2x \sin 2x$
		DAF
		$= \sin x(\cos 4x) + \cos x \, 2\cos 2x \sin 2x$
		DAF
		$= \sin x \cos 4x + \cos x \sin 4x$
		DAF.
		$= \sin(x+4x)$

LS RS

$\sin\left(\frac{\pi}{2}-x\right)\cot\left(\frac{\pi}{2}+x\right)$ $-\sin x$

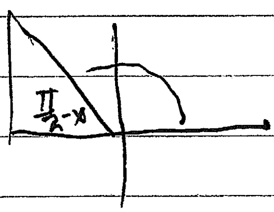
19.

$\cos(x)\left(-\cot\left(\frac{\pi}{2}-x\right)\right)$

$=\cos(x)\left(-(\tan x)\right)$

$= -\cancel{\cos x}\ \frac{\sin x}{\cancel{\cos x}}$

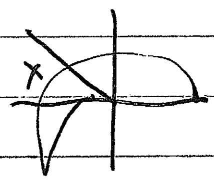
$= -\sin x.$

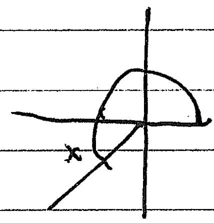


LS	RS

20

LS	RS
$\cos(-x) + \cos(\pi - x)$	$\cos(\pi + x) + \cos x$
$\cos x + (-\cos x)$	$-\cos x + \cos x$
$= 0$	$= 0$





LS ~~RS~~

21. RS

$$\frac{\sin(\pi - x)}{\tan(\pi + x)} \cdot \frac{\cot(\frac{\pi}{2} - x)}{\tan(\frac{\pi}{2} + x)} \, \frac{\cos(2\pi - x)}{\sin(-x)} \qquad \sin x$$

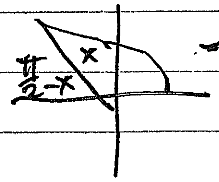
$$\frac{\sin x \cdot \tan x \cdot \cos x.}{(+\tan x)(-\tan(\frac{\pi}{2} - x))(-\sin x)}$$

$$= \frac{\cancel{\sin x}\ \cancel{\tan x}\ \cos x}{+\ \cancel{\tan x}\ \cot x\ \cancel{\sin x}}$$

$$= (\cos x) \div (+\cot(x))$$

$$= \cos x \times \frac{\sin x}{\cos x}$$

$$= \sin x.$$



$-\overset{\tan}{\cancel{\cot}}(\frac{\pi}{2} - x)$

LS R.S.

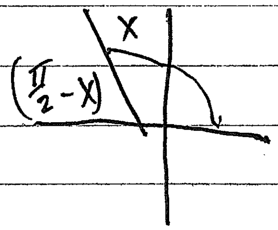
22. $\frac{\sin(-x)}{\sin(\pi+x)} - \frac{\tan(\frac{\pi}{2}+x)}{\cot(x)} + \frac{\cos x}{\sin(\frac{\pi}{2}+x)}$ 3

$= \frac{-\sin x}{-\sin x} - \frac{(-\cot x)}{\cot x} + \frac{\cos x.}{\sin(\frac{\pi}{2}-x)}$

$= 1 + 1 + \frac{\cos x}{\cos x}$

$= 3.$

$\therefore$ LS = RS.



LS RS

23 $\dfrac{\csc(\pi - x)}{\sec(\pi + x)} \cdot \dfrac{\cos(-x)}{\cos(\frac{\pi}{2} + x)}$ $\cot^2 x$

$\dfrac{\csc(x)}{-\sec(x)} \cdot \dfrac{\cos x}{-\cos(\frac{\pi}{2} - x)}$

$\dfrac{\csc(x)}{-\sec x} \cdot \dfrac{\cos x}{-\sin x}$

$\dfrac{\frac{1}{\sin x}}{-\frac{1}{\cos x}} \cdot \dfrac{\cos x}{-\sin x}$

$= -\dfrac{\cos x}{\sin x}\left(-\dfrac{\cancel{\sin x}\cos x}{\sin x}\right)$

$= +\dfrac{\cos^2 x}{\sin^2 x}$

$= \cot^2 x$

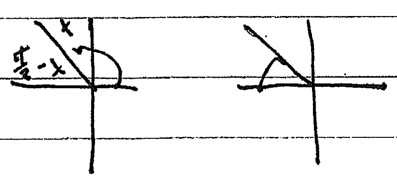
LS RS

24. $\dfrac{\cos(\frac{\pi}{2}+x)\sec(-x)\tan(\pi-x)}{\sec(2\pi+x)\sin(\pi+x)\cot(\frac{\pi}{2}-x)}$ -1

$= \dfrac{\cos(\frac{\pi}{2}-x)(-\sec x)(-\tan x)}{\sec(x)-(\sin x)\tan x}$

$= \dfrac{\sin x \sec x \tan x}{-\sec x \sin x \tan x}$

$= -1$



LS

RS

25 $$\frac{\sin(\pi - x)\cos(\pi + x)\tan(2\pi - x)}{\sec\left(\frac{\pi}{2}+x\right)\csc\left(\frac{3\pi}{2}-x\right)\cot\left(\frac{3\pi}{2}+x\right)}$$

$$\frac{\sin x(-\cos x)(-\tan x)}{\left(-\sec\left(\frac{\pi}{2}-x\right)\right)\left(-\csc\left(\frac{\pi}{2}-x\right)\right)\left(-\cot\left(\frac{\pi}{2}-x\right)\right)}$$

$$= \frac{\sin x \cos x \cancel{\tan x}}{-\csc x \sec x \cancel{\tan x}}$$

$$= \frac{\sin x \cos x}{-\frac{1}{\sin x}\left(\frac{1}{\cos x}\right)}$$

$$= -\sin^2 x \cos^2 x$$

$$\sin^4 x - \sin^2 x$$

$$\cancel{(\sin^2 x)^2 - \sin^2 x}$$

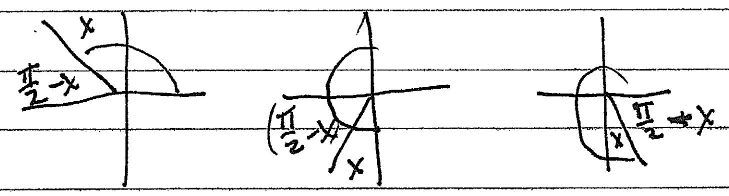
$$\cancel{(1-\cos^2 x)^2 - \sin^2 x}$$

$$\cancel{1 - 2\cos^2 x + \cos^4 x - \sin^2 x}$$

$$= \sin^2 x(\sin^2 x - 1)$$

$$= \sin^2 x(-\cos^2 x)$$

$$= -\sin^2 x \cos^2 x$$



LS RS

2c. $\frac{\sin 2x}{1+\cos 2x}$ $\tan x$

$\frac{2\sin x\cos x}{1+(2\cos^2 x)-1}$

$\frac{\cancel{2}\sin x \cancel{\cos x}}{\cancel{2}\cos^{\cancel{2}} x}$

$= \frac{\sin x}{\cos x}$

$= \tan x$

$\therefore LS = RS$

LS | RS.

27 $\dfrac{1+\cos x}{\sin x}$ | $\cot \frac{x}{2}$

$\dfrac{1+\left(2\cos^2\frac{x}{2}-1\right)}{2\sin\frac{x}{2}\cos\frac{x}{2}}$

$= \dfrac{2\cos^2\frac{x}{2}}{2\sin\frac{x}{2}\cos\frac{x}{2}}$

$= \dfrac{\cos\frac{x}{2}}{\sin\frac{x}{2}}$

$= \cot\frac{x}{2}$.

LS = RS

LS | RS.

28 $2\csc 2x$ | $\sec x \csc x$

$\frac{2}{\sin 2x}$ | $\frac{1}{\cos x} \cdot \frac{1}{\sin x}$

$= \frac{2}{2\sin x \cos x}$ | $\frac{1}{\cos x \sin x}$

$= \frac{1}{\cos x \sin x}$

LS = RS.

LS

RS

29. $2\cot 2x$

$= \dfrac{2}{\tan 2x}$

$= \dfrac{2}{\dfrac{2\tan x}{1-\tan^2 x}}$

$= 2 \times \dfrac{1-\tan^2 x}{2\tan x}$

$= \dfrac{1-\tan^2 x}{2\tan x}$

$\cot x - \tan x$

$= \dfrac{1}{\tan x} - \tan x$

$= \dfrac{1-\tan^2 x}{\tan x}$

LS | RS

30

$$\frac{\cos 2x}{1+\sin 2x}$$

$$\frac{\cos^2 x - \sin^2 x}{(\cos^2 x + \sin^2 x) + 2\sin x\cos x}$$

$$\frac{(\cos x - \sin x)(\cos x + \sin x)}{(\cos x + \sin x)^2}$$

$$= \frac{\cos x - \sin x}{\cos x + \sin x}$$

$$= \frac{\frac{\cos x - \sin x}{\cos x}}{\frac{\cos x + \sin x}{\cos x}}$$

$$= \frac{1 - \tan x}{1 + \tan x}.$$

$$\tan\left(\frac{\pi}{4} - x\right)$$

$$= \frac{\tan\frac{\pi}{4} - \tan x}{1 + \tan\frac{\pi}{4}\tan x}$$

$$= \frac{1 - \tan x}{1 + \tan x}$$

LS | RS

31. $\dfrac{\cos x - \sin x}{\cos x + \sin x}$ | $\sec 2x - \tan 2x$

$$\frac{1}{\cos 2x} - \frac{\sin 2x}{\cos 2x}$$

$$\frac{1 - \sin 2x}{\cos 2x}$$

$$\frac{1 - 2\sin x \cos x}{\cos^2 x - \sin^2 x}$$

$$\frac{\cos^2 x + \sin^2 x - 2\sin x \cos x}{(\cos x - \sin x)(\cos x - \sin x)}$$

$$\frac{(\cos x - \sin x)^{\cancel{2}}}{(\cos x + \sin x)\cancel{(\cos x - \sin x)}}$$

$$\frac{\cos x - \sin x}{\cos x + \sin x}$$

LS $\qquad\qquad$ RS.

32. $\dfrac{1-\cos 2x+\sin 2x}{1+\cos 2x+\sin 2x}$ $\qquad$ $\tan x$

$\qquad\qquad\qquad\qquad\qquad\qquad$ $\dfrac{\sin x}{\cos x}$

$$\frac{1-(1-2\sin^2 x)+2\sin x\cos x}{1+(2\cos^2 x-1)+2\sin x\cos x}$$

$$\frac{2\sin^2 x+2\sin x\cos x}{2\cos^2 x+2\sin x\cos x}$$

$$\frac{2\sin x(\sin x+\cos x)}{2\cos x(\sin x+\cos x)}$$

$$=\frac{\sin x}{\cos x}$$

LS = RS.

LS RS

33 $\cos^6 x - \sin^6 x$

$\cos 2x \left(1 - \frac{1}{4}\sin^2 2x\right)$

$\cos 2x \left(1 - \frac{1}{2}\sin 2x\right)\left(1 + \frac{1}{2}\sin 2x\right)$

$\cos 2x \left(1 - \frac{1}{2}(2\sin x\cos x)\right)\left(1 + \frac{1}{2}(2\sin x\cos x)\right)$

$\cos 2x \left(1 - \sin^2 x\cos^2 x\right)$

$\cancel{\left(1 - 2\sin^2 x\right)\left(1 - \sin^2 x\cos^2 x\right)}$

$\cancel{1 - 2\sin^2 x - \sin^2 x\cos^2 x + 2\sin^4 x\cos^2 x}$

LS

$\left(\cos^2 x - \sin^2 x\right)\left(\cos^4 x + \cos^2 x\sin^2 x + \sin^4 x\right)$

$\cos 2x \left(\cos^4 x + \cos^2 x\sin^2 x + \sin^4 x\right)$

$\cos 2x \left(\cos^4 x + 2\cos^2 x\sin^2 x + \sin^4 x - \sin^2 x\cos^2 x\right)$

$\cos 2x \left(\left(\cos^2 x + \sin^2 x\right)^2 - \sin^2 x\cos^2 x\right)$

$\cos 2x \left(1^2 - \sin^2 x\cos^2 x\right)$

$\cos 2x \left(1 - \sin^2 x\cos^2 x\right)$

$\therefore$ LS = RS.

RS | RS

34 $4(\cos^6 x + \sin^6 x)$

$= 4(\cos^2 x + \sin^2 x)(\cos^4 x - \cos^2 \sin^2 x + \sin^4 x)$

$= 4(1)(\cos^4 x - \cos^2 x \sin^2 x + \sin^4 x)$

$= 4((1-\sin^2 x)^2 - (1-\sin^2 x)\sin^2 x + \sin^4 x)$

$= 4$

$= 4\left[(1 - 2\sin^2 x + \sin^4 x) - (\sin^2 x - \sin^4 x) + \sin^4 x\right]$

$= 4\left[1 - 3\sin^2 x + 3\sin^4 x\right].$

$1 + 3\cos^2 2x$

$= 1 + 3(1 - 2\sin^2 x)^2$

$= 1 + 3(1 - 4\sin^2 x + 4\sin^4 x)$

$= 1 + 3 - 12\sin^2 x + 12\sin^4 x$

$= 4 - 12\sin^2 x + 12\sin^4 x$

$4(1 - 3\sin^2 x + 3\sin^4 x)$

35

LS

$$\sec x - \tan x$$

$$\frac{1}{\cos x} - \frac{\sin x}{\cos x}$$

$$\frac{1-\sin x}{\cos x}$$

$$\frac{1 - 2\sin\frac{x}{2}\cos\frac{x}{2}}{\cos^2\frac{x}{2} - \sin^2\frac{x}{2}}$$

$$\frac{\cos^2\left(\frac{x}{2}\right) + \sin^2\left(\frac{x}{2}\right) - 2\sin\frac{x}{2}\cos\frac{x}{2}}{\left(\cos\frac{x}{2} - \sin\frac{x}{2}\right)\left(\cos\frac{x}{2} + \sin\frac{x}{2}\right)}$$

$$\frac{\left(\cos\frac{x}{2} - \sin\frac{x}{2}\right)^2}{\left(\cos\frac{x}{2} - \sin\frac{x}{2}\right)\left(\cos\frac{x}{2} + \sin\frac{x}{2}\right)}$$

$$\frac{\cos\frac{x}{2} - \sin\frac{x}{2}}{\cos\frac{x}{2} + \sin\frac{x}{2}}$$

RS

$$\tan\left(\frac{\pi}{4} - \frac{x}{2}\right)$$

$$\cancel{= \tan\tfrac{1}{2}\left(\tfrac{\pi}{2} - x\right)}$$

$$= \frac{\tan\frac{\pi}{4} - \tan\frac{x}{2}}{1 + \tan\frac{\pi}{4}\tan\frac{x}{2}}$$

$$= \frac{1 - \tan\frac{x}{2}}{1 + \tan\frac{x}{2}}$$

$$= \frac{1 - \frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}}{1 + \frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}}$$

$$= \frac{\cos\frac{x}{2} - \sin\frac{x}{2}}{\cos\frac{x}{2}}\left(\frac{\cos\frac{x}{2}}{\cos\frac{x}{2} + \sin\frac{x}{2}}\right)$$

$$= \frac{\cos\frac{x}{2} - \sin\frac{x}{2}}{\cos\frac{x}{2} + \sin\frac{x}{2}}$$

$\therefore$ LS = RS.

36 $\dfrac{\sin 2x}{1+\cos 2x}\left(\dfrac{\cos x}{1+\cos x}\right)$ $\tan \frac{x}{2}$.

$\dfrac{\sin \frac{x}{2}}{\cos \frac{x}{2}}$.

$\dfrac{2 \sin x \cos x \cos x}{\left(1+\left(2\cos^2 x - 1\right)\right)\left(1+\cos x\right)}$.

$= \dfrac{2 \sin x \cos^2 x}{2 \cos^2 x \left(1+\cos x\right)}$

$= \dfrac{\cancel{2} \sin x}{1+\cos x}$.

$= \dfrac{\cancel{2}\left(2 \sin\frac{x}{2} \cos\frac{x}{2}\right)}{1+\left(2\cos^2\frac{x}{2}-1\right)}$

$= \dfrac{\cancel{2} \sin\frac{x}{2}\cos\frac{x}{2}}{2 \cos^2\left(\frac{x}{2}\right)}$.

$= \dfrac{\sin\frac{x}{2}}{\cos\frac{x}{2}}$

$\therefore$ LS = RS.

LS RS

37 $\sin^2 x + \cos^4 x$ | $\cos^2 x + \sin^4 x.$

$\sin^2 x + (1-\sin^2 x)^2.$ | $= 1-\sin^2 x + \sin^4 x.$

$\sin^2 x + 1 - 2\sin^2 x + \sin^4 x$

$1 - \sin^2 x + \sin^4 x$

LS = RS

LS RS

38

$\tan x - \cot x$

$(\tan x - 1)(\cot x + 1)$

$(\tan x - 1)\left(\frac{1}{\tan x} + 1\right)$

$\frac{\tan x}{\tan x} - \frac{1}{\tan x} + \tan x - 1.$

$1 - \frac{1}{\tan x} + \tan x - 1$

$-\cot x + \tan x$

LS = RS

LS $\qquad\qquad\qquad\qquad$ RS.

39. $\cos x \qquad\qquad \sin x \tan^2 x \cot^3 x$

$$\sin x \ \frac{\sin^2 x}{\cos^2 x} \ \frac{\cos^3 x}{\sin^3 x}$$

$$= \cos x$$

$$LS = RS$$

LS RS.

40. $(\sin x + \cos x)(\tan x + \cot x)$ $\quad\quad$ $\sec x + \csc x$

$$= (\sin x + \cos x)\left(\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}\right) \quad\quad = \frac{1}{\cos x} + \frac{1}{\sin x}$$

$$= \frac{\sin x + \cos x}{\cos x \sin x}$$

$$= \frac{\sin^2 x}{\cos x} + \frac{\sin x \cos x}{\sin x} + \frac{\cos x \sin x}{\cos x} + \frac{\cos^2 x}{\sin x}$$

$$= \frac{\sin^2 x}{\cos x} + \cos x + \sin x + \frac{\cos^2 x}{\sin x}$$

$$= \frac{\sin^3 x + \cos^2 x \sin x + \cos x \sin^2 x + \cos^3 x}{\cos x \sin x}$$

$$= \frac{\sin x(\sin^2 x + \cos^2 x) + \cos x(\sin^2 x + \cos^2 x)}{\cos x \sin x}$$

$$= \frac{\sin x(1) + \cos x(1)}{\cos x \sin x}$$

$\therefore$ LS = RS

LS	RS
41 $\sin^4 x + \cos^4 x$	$\sin^2 x\,(\csc^2 x - 2\cos^2 x)$
$= \underbrace{\sin^4 x + 2\sin^2 x\cos^2 x + \cos^4 x} - 2\sin^2 x\cos^2 x$	$= \sin^2 x\left(\frac{1}{\sin^2 x} - 2\cos^2 x\right)$
$= (\sin^2 x + \cos^2 x)^2 - 2\sin^2 x\cos^2 x$	$= \frac{\sin^2 x}{\sin^2 x} - 2\cos^2 x\sin^2 x$
$= 1^2 - 2\sin^2 x\cos^2 x$	$= 1 - 2\cos^2 x\sin^2 x$
$= 1 - 2\sin^2 x\cos^2 x$	

LS = RS.

LS $\qquad\qquad\qquad\qquad$ RS.

42 $\sin^3 x + \cos^3 x \qquad\qquad (1 - \sin x \cos x)(\sin x + \cos x)$

$(\sin x + \cos x)(\sin^2 x - \sin x \cos x + \cos^2 x)$

$(\sin x + \cos x)(1 - \sin x \cos x)$

$\therefore LS = RS$

LS — RS

43 $\cos\left(\frac{\pi}{12} - x\right)\sec\left(\frac{\pi}{12}\right) - \sin\left(\frac{\pi}{12} - x\right)\csc\frac{\pi}{12}$ — $4\sin x.$

$= \cancel{\cos\frac{\pi}{12}\sin x -}$

$= \left(\cos\frac{\pi}{12}\cos x - \sin\frac{\pi}{12}\sin x\right)\left(\frac{1}{\cos\frac{\pi}{12}}\right) - \left(\sin\frac{\pi}{12}\cos x - \sin x\cos\frac{\pi}{12}\right)\frac{1}{\sin\frac{\pi}{12}}$

$= \frac{\cos\frac{\pi}{12}\cos x}{\cos\frac{\pi}{12}} - \frac{\sin\frac{\pi}{12}\sin x}{\cos\frac{\pi}{12}} - \frac{\sin\frac{\pi}{12}\cos x}{\sin\frac{\pi}{12}} + \frac{\sin x\cos\frac{\pi}{12}}{\sin\frac{\pi}{12}}$

$= \cos x - \tan\frac{\pi}{12}\sin x - \cos x + \sin x\cot\frac{\pi}{12}$

$= \sin x\left(\cot\frac{\pi}{12} - \tan\frac{\pi}{12}\right)$

$= \sin x\left(\frac{1}{\tan\frac{\pi}{12}} - \tan\frac{\pi}{12}\right)$

$= \sin x\left(\frac{1 - \tan^2\frac{\pi}{12}}{\tan\frac{\pi}{12}}\right)$ ← almost double angle (reciprical of $\tan 2\theta$)

$\cancel{= \sin x\left(\tan 2\frac{\pi}{12}\right)}$

$\cancel{= \sin x\left(\tan\frac{\pi}{6}\right)}$

$\cancel{= \sin x \cdot}$

$= \sin x(2)\left(\frac{1 - \tan^2\frac{\pi}{12}}{2\tan\frac{\pi}{12}}\right)$

$= \sin x \, 2 \, \frac{1}{\tan 2\left(\frac{\pi}{12}\right)}$

$= \sin x(2)\left(\frac{1}{\frac{1}{2}}\right)$

$= 4\sin x$

LS = RS

LS $\qquad\qquad\qquad\qquad$ RS.

44 $\tan(x-y) + \tan(y-z)$ $\qquad$ $\dfrac{\sec^2 y\,(\tan x - \tan z)}{(1+\tan x \tan y)(1+\tan y \tan z}$

$$= \frac{\tan x - \tan y}{1 + \tan x \tan y} + \frac{\tan y - \tan z}{1 + \tan y \tan z}$$

common denominator $$= \frac{(\tan x - \tan y)(1 + \tan y \tan z) + (\tan y - \tan z)(1 + \tan x \tan y)}{(1 + \tan x \tan y)(1 + \tan y \tan z)}$$

$$= \frac{(\tan x - \tan y + \cancel{\tan x \tan y \tan z} - \tan^2 y \tan z) + (\tan y - \tan z + \tan^2 y \tan x + \cancel{\tan x \tan y \tan z})}{(1 + \tan x \tan y)(1 + \tan y \tan z)}$$

$$= \frac{\tan x - \tan^2 y \tan x - \tan z + \tan^2 y \tan z}{(1 + \tan x \tan y)(1 + \tan y \tan z)}$$

$$= \frac{\tan x(1 - \tan^2 y) - \tan z\,(1 - \tan^2 y)}{(1 + \tan x \tan y)(1 + \tan y \tan z)}$$

$$= \frac{(\tan x - \tan z)(\sec^2 y)}{(1 + \tan x \tan y)(1 + \tan y \tan z)}$$

$\therefore$ LS = RS

	LS	RS
4S	$\sin 8x$	$8 \sin x \cos x \cos 2x \cos 4x$

$= 2(\sin 4x)\cos 4x$

$= 2(2\sin 2x \cos 2x)\cos 4x$

$= 2(2(2\sin x \cos x)(\cos 2x \cos 4x))$

$= 8 \sin x \cos x \cos 2x \cos 4x$

LS = RS.

LS	RS

46 $\sin x$ $\quad$ $1 - 2\sin^2\left(\frac{\pi}{4} - \frac{x}{2}\right)$

$= \cos 2\left(\frac{\pi}{4} - \frac{x}{2}\right)$

$= \cos\left(\frac{\pi}{2} - x\right)$

$= \sin x$

$LS = RS$

LS RS

47 $\sin(x+y) + \sin(x-y)$ $2\sin x \cos y$.

$= \sin x \cos y + \cos x \sin y + \sin x \cos y - \cos x \sin y$.

$= 2 \sin x \cos y$.

LS = RS.

LS | RS

$$48.\quad \frac{\sin(x-y)}{\sin x \sin y} + \frac{\sin(y-z)}{\sin y \sin z} + \frac{\sin(z-x)}{\sin z \sin x} \quad \Big| \quad 0$$

$$= \frac{\sin x \cos y - \cos x \sin y}{\sin x \sin y} + \frac{\sin y \cos z - \cos z \sin y}{\sin y \sin z} + \frac{\sin z \cos x - \cos z \sin}{\sin z \sin x}$$

common denominator

$$= \frac{\sin x \cos y \sin z - \cos x \sin y \sin z + \sin y \cos z \sin x - \cos y \sin z \sin x + \sin z \cos x \sin y - \cos z \sin x}{\sin x \sin y \sin z}$$

$$= \frac{0}{\sin x \sin y \sin z}$$

$\therefore$ LS = RS

LS RS

49. $\tan x + \tan(\pi - x) + \cot\left(\frac{\pi}{2} + x\right)$ $\qquad$ $\tan(2\pi - x)$

$= \tan x - \tan x + \left(-\cot\left(\frac{\pi}{2} - x\right)\right)$ $\qquad$ $-\tan x$

$= 0 - \cot\left(\frac{\pi}{2} - x\right)$

$= -\tan x$

$\therefore LS = RS$

LS	RS
So $\sin\left(\frac{\pi}{2}+x\right)\cos\left(\frac{\pi}{2}-x\right)\cot\left(\frac{3\pi}{2}+x\right)$	$\sin\left(\frac{\pi}{2}-x\right)\sin\left(\frac{3\pi}{2}-x\right)\cot\left(\frac{\pi}{2}+x\right)$
$=\sin\left(\frac{\pi}{2}-x\right)(-\cos x)\left(-\cot\left(\frac{\pi}{2}-x\right)\right)$	$\cos x\left(-\sin\left(\frac{\pi}{2}-x\right)\right)\left(-\cot\left(\frac{\pi}{2}-x\right)\right)$
$=\cos x(-\cos x)(-\tan x)$	$\cos x(-\cos x)(-\tan x)$
$=\cos^2 x\tan x$	$\cos^2 x\tan x$

$\therefore LS = RS$

	LS	RS
S1.	$\tan\left(\frac{\pi}{2}-x\right)-\cot\left(\frac{3\pi}{2}-x\right)+\tan(2\pi-x)-\cot(\pi-x)$	$\dfrac{4-2\sec^2 x}{\tan x}$
	$\cot x - \cot\left(\frac{\pi}{2}-x\right)+(-\tan x)-(-\cot x)$	

$= \cot x - \tan x - \tan x + \cot x$

$= 2\cot x - 2\tan x$

$= \dfrac{2}{\tan x} - 2\tan x$

$= \dfrac{2-2\tan^2 x}{\tan x}$

$= 2 - 2(\sec^2 x - 1)$

$= 2 + 2 - 2\sec^2 x$

$= 4 - 2\sec^2 x$

$\therefore LS = RS$

LS | RS

52 $\tan(x+y+z)$ | $\dfrac{\tan x+\tan y+\tan z-\tan x\tan y\tan z}{1-\tan x\tan y-\tan x\tan z-\tan y\tan z}$

$$= \frac{\tan(x+y)+\tan(z)}{1-\tan(x+y)\tan(z)}$$

$$= \frac{\left(\frac{\tan x+\tan y}{1-\tan x\tan y}\right)+\tan z.}{1-\left(\frac{\tan x+\tan y}{1-\tan x\tan y}\right)\tan z}$$

Com Den
$$= \frac{\frac{\tan x+\tan y+(\tan z-\tan x\tan y\tan z)}{1-\tan x\tan y}}{\frac{(1-\tan x\tan y)-\tan x\tan z-\tan y\tan z}{1-\tan x\tan y}}$$

invert 'n' multiply
$$= \frac{\tan x+\tan y+\tan z-\tan x\tan y\tan z.}{(1-\tan x\tan y)}\left(\frac{(1-\tan x\tan y)}{1-\tan x\tan y-\tan x\tan z-\tan y\tan z}\right)$$

$$= \frac{\tan x+\tan y+\tan z-\tan x\tan y\tan z}{1-\tan x\tan y-\tan x\tan z-\tan y\tan z}$$

LS | RS

53 $\csc^2\left(\frac{\pi}{2}-x\right)$ | $1 + \sin^2 x \csc^2\left(\frac{\pi}{2}-x\right)$

$\sec^2 x$ | $= 1 + \sin^2 x \sec^2(x)$

$\frac{1}{\cos^2 x}$ | $= 1 + \frac{\sin^2 x}{\cos^2 x}$

$\cancel{1 + \tan^2 x}$

$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x}$

$= \frac{1}{\cos^2 x}$

$\therefore LS = RS$

LS | RS.

54 $\tan\left(\frac{\pi}{4}+x\right)+\tan\left(\frac{\pi}{4}-x\right)$ $\qquad$ $2\sec 2x$.

$\frac{2}{\cos 2x}$

$$\frac{\tan\frac{\pi}{4}+\tan x}{1-\tan\frac{\pi}{4}\tan x}+\frac{\tan\frac{\pi}{4}-\tan x}{1+\tan\frac{\pi}{4}\tan x}$$

$$\frac{1+\tan x}{1-\tan x}+\frac{1-\tan x}{1+\tan x}$$

com denom. $=\frac{(1+\tan x)(1+\tan x)+(1-\tan x)(1-\tan x)}{(1-\tan x)(1+\tan x)}$

$$=\frac{(1+\tan x+\tan x+\tan^2 x)+(1-\tan x-\tan x+\tan^2 x)}{1-\tan x+\tan x-\tan^2 x}$$

$$=\frac{2+2\tan^2 x}{1-\tan^2 x}$$

$$=\frac{2(1+\tan^2 x)}{(1-\tan^2 x)}$$

$$=\frac{2\left(1+\frac{\sin^2 x}{\cos^2 x}\right)}{1-\frac{\sin^2 x}{\cos^2 x}}$$

$$=2\left(\frac{\cos^2 x+\sin^2 x}{\cos^2 x}\right)\frac{(\cos^2 x)}{(\cos^2 x-\sin^2 x)}$$

$$=\frac{2(1)}{\cos^2 x-\sin^2 x}$$

$$=\frac{2}{\cos 2x}.$$

so LS = RS

LS | RS

55.

$$\frac{1-\sin 2x}{\cos 2x}$$

$$= \frac{1-2\sin x\cos x}{\cos^2 x-\sin^2 x}$$

$$= \frac{\cos^2 x+\sin^2 x-2\sin x\cos x}{(\cos x-\sin x)(\cos x+\sin x)}$$

$$= \frac{(\cos x-\sin x)^2}{(\cos x-\sin x)(\cos x+\sin x)}$$

$$= \frac{\cos x-\sin x}{\cos x+\sin x}$$

RS

$$\frac{\cos 2x}{1+\sin 2x}$$

$$= \frac{\cos^2 x-\sin^2 x}{1+2\cos x\sin x}$$

$$= \frac{(\cos x-\sin x)(\cos x+\sin x)}{\cos^2 x+\sin^2 x+2\cos x\sin x}$$

$$= \frac{(\cos x-\sin x)(\cos x+\sin x)}{(\cos x+\sin x)^2}$$

$$= \frac{\cos x-\sin x}{\cos x+\sin x}$$

$\therefore$ LS = RS

LS RS

56 $\dfrac{\sin 4x}{1-\cos 4x}\cdot\dfrac{1-\cos 2x}{\cos 2x}$ $\tan x$

$$\frac{(2\sin 2x\cos 2x)(1-\cos 2x)}{(1-(1-2\sin^2 2x))\cos 2x}$$

$$=\frac{(2\sin 2x)(1-\cos 2x)}{(2\sin^2 2x)\cancel{(\cos 2x)}}$$

$$=\frac{1-\cos 2x}{\sin 2x}$$

$$=\frac{1-(1-2\sin^2 x)}{2\cos x\sin x}$$

$$=\frac{2\sin^2 x}{2\cos x\sin x}$$

$$=\frac{\sin x}{\cos x}$$

$$=\tan x$$

$\therefore$ LS = RS.