# Practice Problems For Finding Unknown Coefficients.

#1) $f(x) = ax^3 + bx^2 + cx + d$

Critical Points: $x=2$, $x=-1$, $f(0) = 1$, $f'(0) = 6$.

i) find d

$f(0) = a(0)^3 + b(0)^2 + c(0) + d$

$1 = 0 + 0 + 0 + d$

$\boxed{d = 1}$

ii) find derivatives

$f'(x) = 3ax^2 + 2bx + c$

$f''(x) = 6ax + 2b$

iii) use $f'(0) = 6$

$f'(0) = 3a(0)^2 + 2b(0) + c$

$6 = 0 + 0 + c$

$\boxed{c = 6}$

iv) set $f'(2) = 0$

$0 = 3a(2)^2 + 2b(2) + c$

$0 = 12a + 4b + 6$

v) set $f'(-1) = 0$

$0 = 3a(-1)^2 + 2b(-1) + c$

$0 = 3a - 2b + 6$.

vi) elimination

$0 = 12a + 4b + 6$

$0 = 6a - 4b + 12$

$0 = 18a + 18$.

vii) find a

$-18 = 18a$

$\boxed{-1 = a}$

viii) find b.

$0 = 3(-1) - 2b + 6$.

$-3 = -2b$

$\boxed{b = \frac{3}{2}}$

$\therefore a = -1, b = \frac{3}{2}, c = 6, d = 1$

$f(x) = -x^3 + \frac{3}{2}x^2 + 6x + 1$

#2) $f(x) = \frac{1}{3}x^3 + bx^2 + c$ Critical Point: $(3, 2)$

i) find derivatives

$f'(x) = x^2 + 2bx$

$f''(x) = 2x + 2b$

ii) set $f'(3) = 0$.

$0 = (3)^2 + 2b(3)$

$= 9 + 6b$

$\boxed{b = -\frac{3}{2}}$

iii) sub in $(3, 2)$ and b to find c

$2 = \frac{1}{3}(3)^3 + \left(-\frac{3}{2}\right)(3)^2 + c$

$2 = 9 - 13.5 + c$

$\boxed{c = 6.5}$

$\therefore b = -\frac{3}{2}$, $c = 6.5$

$f(x) = \frac{1}{3}x^3 - \frac{3}{2}x^2 + 6.5$.

#3) $f'(x) = ax^2 + bx + c$ max: $x = 3$ min: $x = -5$ $f''(3) = 8$.

i) find d. derivative

$f''(x) = 2ax + b$.

$8 = 2a(3) + b$.

$8 = 6a + b$

ii) use max = 3

$0 = a(3)^2 + b(3) + c$

$0 = 9a + 3b + c$

iii) use min $x = -5$

$0 = a(-5)^2 + b(-5) + c$

$0 = 25a - 5b + c$.

iv) combine ii) and iii)

$-9a - 3b = -25a + 5b$.

$16a - 8b = 0$.

v) elimination

$0 = 16a - 8b + 0$

$0 = 48a + 8b + 64$

$64a + 64$

vi) find a

$0 = 64a + 64$.

$\boxed{a = -1}$

$\therefore a = -1, b = -2, c = 15$.

vii) find b

$16(-1) - 8b = 0$

$\boxed{b = -2}$

viii) $0 = 9(-1) + 3(-2) + c$

$0 = -9 - 6 + c$

$\boxed{c = 15}$

$f'(x) = -x^2 - 2x + 15$

#4) $P(x) = ax^3 + bx^2 + cx + 1$ P.O.I: $x = 2$ min: $x = -2$ $f(1) = 2$

i) use $f(1) = 2$

$2 = a(1)^3 + b(1)^2 + c(1) + 1$

$2 = a + b + c + 1$

$1 = a + b + c$

ii) find the derivatives

$f'(x) = 3ax^2 + 2bx + c$

$f''(x) = 6ax + 2b$

iii) min: $x = -2$

$0 = 3a(-2)^2 + 2b(-2) + c$

$0 = 12a - 4b + c$

iv) sub in the P.O.I

$0 = 6a(2) + 2b$

$0 = 12a + 2b$

v) combine i) and iii)

$12a - 4b = a + b - 1$

$11a - 5b = -1$

v) elimination

$-2 = 22a - 10b$

$0 = 60a + 10b$

$-2 = 82a$.

vi) find a

$\frac{-2}{82} = a$

$\boxed{a = -\frac{1}{41}}$

vii) find b

$0 = 12\left(-\frac{1}{41}\right) + 2b$

$\frac{12}{41} \times \frac{1}{2} = b$. $\boxed{b = \frac{6}{41}}$

viii) find c

$1 = \left(-\frac{1}{41}\right) + \left(\frac{6}{41}\right) + c$.

$\frac{41}{41} + \frac{1}{41} - \frac{6}{41} = c$.

$\boxed{c = \frac{36}{41}}$

$\therefore a = -\frac{1}{41}, b = \frac{6}{41}, c = \frac{36}{41}$

$f(x) = -\frac{1}{41}x^3 + \frac{6}{41}x^2 + \frac{36}{41}x + 1$