Exercises 1.4

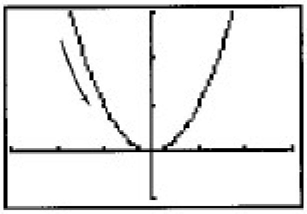
1. Graph (c). Window: $[-4, 4]$ by $[-3, 3]$, $0 \le t \le 2\pi$

2. Graph (a) Window: $[-2, 2]$ by $[-2, 2]$, $0 \le t \le 2\pi$

3. Graph (d). Window: $[-10, 10]$ by $[-10, 10]$, $0 \le t \le 2\pi$

4. Graph (b). Window: $[-15, 15]$ by $[-15, 15]$, $0 \le t \le 2\pi$

5. (a)

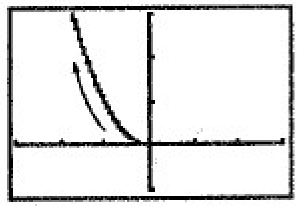


$[-3, 3]$ by $[-1, 3]$

No initial or terminal point

(b) $y = x^2$; all

6. (a)

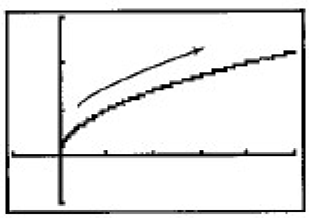


$[-3, 3]$ by $[-1, 3]$

Initial point: $(0, 0)$
Terminal point: None

(b) $y = x^2$; left half (or $x = -\sqrt{y}$; all)

7. (a)

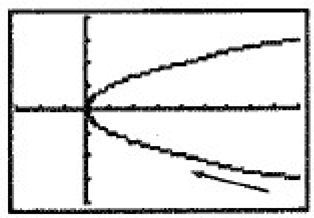


$[-1, 5]$ by $[-1, 3]$

Initial point: $(0, 0)$
Terminal point: None

(b) $y = \sqrt{x}$; all (or $x = y^2$; upper half)

8. (a)

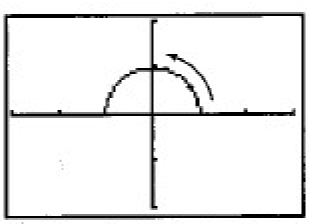


$[-3, 9]$ by $[-4, 4]$

No initial or terminal point

(b) $x = y^2$; all

9. (a)

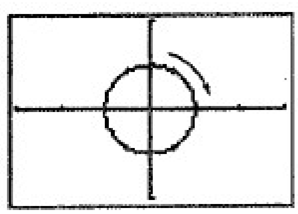


$[-3, 3]$ by $[-2, 2]$

Initial point: $(1, 0)$
Terminal point: $(-1, 0)$

(b) $x^2 + y^2 = 1$; upper half (or $y = \sqrt{1 - x^2}$; all)

10. (a)

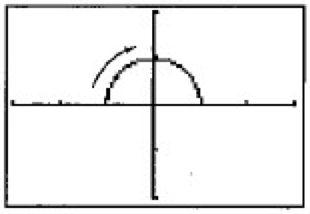


$[-3, 3]$ by $[-2, 2]$

Initial and terminal point: $(0, 1)$

(b) $x^2 + y^2 = 1$; all

11. (a)

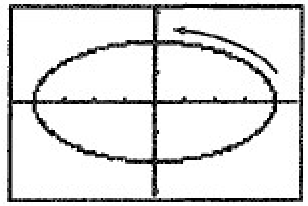


[–3, 3] by [–2, 2]

Initial point: $(-1, 0)$
Terminal point: $(0, 1)$

(b) $x^2 + y^2 = 1$; upper half (or $y = \sqrt{1 - x^2}$; all)

12. (a)

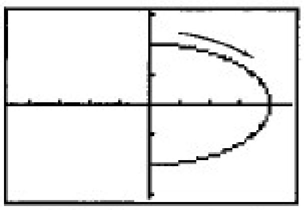


[−4.7, 4.7] by [−3.1, 3.1]

Initial and terminal point: $(4, 0)$

(b) $\left(\frac{x}{4}\right)^2 + \left(\frac{x}{2}\right)^2 = 1$; all

13. (a)



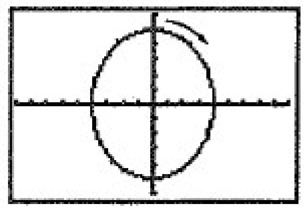
[–4.7, 4.7] by [–3.1, 3.1]

Initial point: $(0, 2)$
Terminal point: $(0, -2)$

(b) $\left(\frac{x}{4}\right)^2 + \left(\frac{y}{2}\right)^2 = 1$; right half (or $x = 2\sqrt{4 - y^2}$; all)

(or $x = 2\sqrt{4 - y^2}$; all)

14. (a)

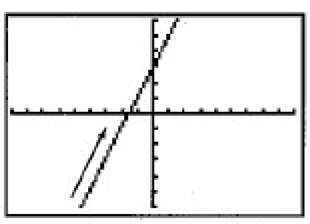


[−9, 9] by [−6, 6]

Initial and terminal point: $(0, 5)$

(b) $\left(\frac{x}{4}\right)^2 + \left(\frac{y}{5}\right)^2 = 1$; all

15. (a)

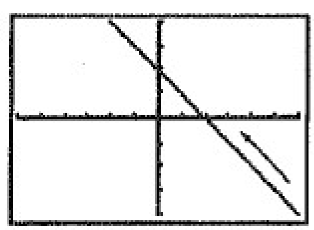


[–9, 9] by [–6, 6]

Initial and terminal point: $(0, 5)$

(b) $y = 2x + 3$; all

16. (a)

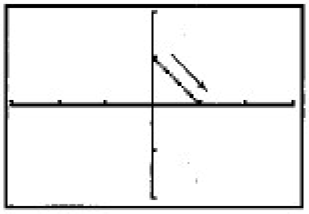


[−6, 6] by [−4, 4]

No initial or terminal point

(b) $y = -x + 2$; all

17. **(a)**

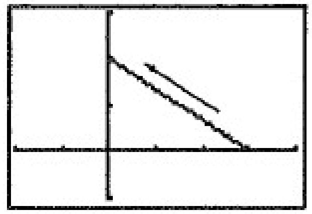


[–3, 3] by [–2, 2]

Initial point: $(0, 1)$
Terminal point: $(1, 0)$

(b) $y = -x + 1$; $(0, 1)$ to $(1, 0)$

18. **(a)**

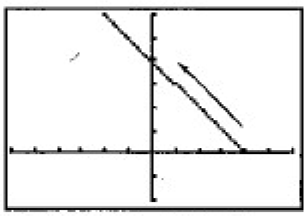


[−2, 4] by [−1, 3]

Initial point: $(3, 0)$
Terminal point: $(0, 2)$

(b) $y = -\frac{2}{3}x + 2$; $(3, 0)$ to $(0, 2)$

19. **(a)**

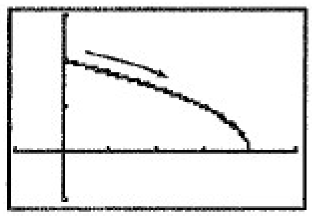


[–6, 6] by [–2, 6]

Initial point: $(4, 0)$
Terminal point: None

(b) $y = -x + 4$; $x \le 4$

20. **(a)**

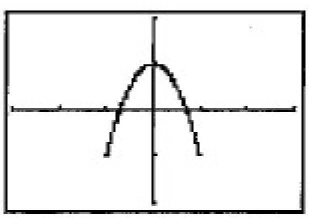


[−1, 5] by [−1, 3]

Initial point: $(0, 2)$
Terminal point: $(4, 0)$

(b) $y = \sqrt{4 - x}$; $x \ge 0$

21. **(a)**

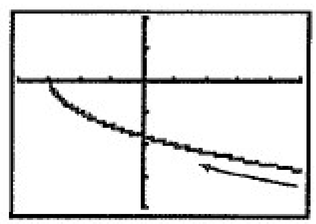


[–3, 3] by [–2, 2]

The curve is traced and retraced in both directions, and there is no initial or terminal point.

(b) $y = -2x^2 + 1$; $-1 \le x \le 1$

22. **(a)**



[−4, 5] by [−4, 2]

Initial point: None
Terminal point: $(-3, 0)$

(b) $x = y^2 - 3$; lower half (or $y = -\sqrt{x + 3}$; all

23. Possible answer: $x = -1 + 5t, y = -3 + 4t, 0 \leq t \leq 1$

24. Possible answer: $x = -1 + 4t, y = 3 - 5t, 0 \leq t \leq 1$

25. Possible answer: $x = t^2 + 1, y = t, t \leq 0$

26. Possible answer: $x = t, y = t^2 + 2t, t \leq -1$

27. Possible answer: $x = 2 - 3t, y = 3 - 4t, t \geq 0$

28. Possible answer: $x = -1 + t, y = 2 - 2t, t \geq 0$

29. $1 < t < 3$ **30.** $3 < t \leq 5$ **31.** $-5 \leq t < -3$ **32.** $-3 < t < 1$

33. Possible answer: $x = t, y = t^2 + 2t + 2, t > 0$

34. Possible answer: $x = t, y = \sqrt{t + 3}, t > 0$

35. Possible answers:

(a) $x = a \cos t, y = -a \sin t, 0 \leq t \leq 2\pi$

(b) $x = a \cos t, y = a \sin t, 0 \leq t \leq 2\pi$

(c) $x = a \cos t, y = -a \sin t, 0 \leq t \leq 4\pi$

(d) $x = a \cos t, y = a \sin t, 0 \leq t \leq 4\pi$

36. Possible answers:

(a) $x = -a \cos t, y = b \sin t, 0 \leq t \leq 2\pi$

(b) $x = -a \cos t, y = -b \sin t, 0 \leq t \leq 2\pi$

(c) $x = -a \cos t, y = b \sin t, 0 \leq t \leq 4\pi$

(d) $x = -a \cos t, y = -b \sin t, 0 \leq t \leq 4\pi$

37. False. It is an ellipse.

38. True. Circle starting at $(2, 0)$ and ending at $(2, 0)$.

39. D **40.** C **41.** A **42.** E

43. **(a)** The resulting graph appears to be the right half of a hyperbola in the first and fourth quadrants. The parameter a determines the x-intercept. The parameter b determines the shape of the hyperbola. If b is smaller, the graph has less steep slopes and appears "sharper." If b is larger, the slopes are steeper and the graph appears more "blunt."

(b) This appears to be the left half of the same hyperbola.

(c) Because both sec t and tan t are discontinuous at these points. This might cause the grapher to include extraneous lines (the asymptotes to the hyperbola) in its graph.

(d) $\left(\frac{x}{a}\right)^2 - \left(\frac{y}{b}\right)^2 = (\sec t)^2 - (\tan t)^2 = 1$

by a standard trigonometric identity.

(e) This changes the orientation of the hyperbola. In this case, b determines the y-intercept of the hyperbola, and a determines the shape. The parameter interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ gives the upper half of the hyperbola. The parameter interval $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$ gives the lower half. The same values of t cause discontinuities and may add extraneous lines to the graph.

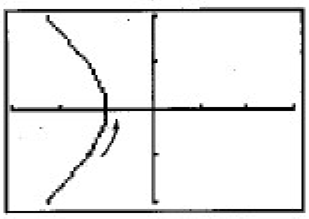
44. (a) h determines the x-coordinate of the center of the circle. It causes a horizontal shift of the graph.

(b) k determines the y-coordinate of the center of the circle. It causes a vertical shift of the graph.

(c) $x = 5\cos t + 2, y = 5\sin t - 3, 0 \leq t \leq 2\pi$

(d) $x = 5\cos t - 3, y = 2\sin t + 4, 0 \leq t \leq 2\pi$

45. (a)



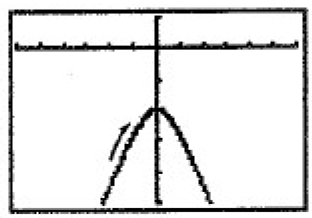
[–3, 3] by [–2, 2]

No initial or terminal point

(b) $x^2 - y^2 = 1$; left branch

(or $x = -\sqrt{y^2 + 1}$; all)

46. (a)



[−6, 6] by [−5, 1]

No initial or terminal point

(b) $\left(\frac{y}{2}\right)^2 - x^2 = 1$; lower branch

(or $y = -2\sqrt{x^2 + 1}$; all)

47. $x = 2\cot t, y = 2\sin^2 t, 0 < t < \pi$

48. (a) If $x_2 = x_1$ then the line is a vertical line and the first parametric equation gives $x = x_1$, while the second will give all real values for y since it cannot be the case that $y_2 = y_1$ as well. Otherwise, solving the first equation for t gives

$$t = \frac{x - x_1}{x_2 - x_1}.$$

Substituting that into the second equation for t gives

$$y = y_1 + \left[\frac{y_2 - y_1}{x_2 - x_1}\right](x - x_1)$$

which is the point-slope form of the equation for the line through (x_1, y_1) and (x_2, y_2). Note that the first equation will cause x to take on all real values, because $x_2 - x_1$ is not zero. Therefore, all of the points on the line will be traced out.

(b) Use the equations for x and y given in part (a) with $0 \leq t \leq 1$.