DOUBLE ANGLE FORMULAS
SUPPLEMENTARY PROBLEMS

1. Use a Double Angle Formula to rewrite each expression.

(a) $\cos 2(2x)$ (b) $\sin 3x$ (c) $\tan 6x$

(d) $\sin \frac{1}{2}x$ (e) $\cos \frac{2}{3}x$ (f) $\tan(-7x)$

2. Express as a single sine or cosine function.

(a) $2 \sin 3\theta \cos 3\theta$ (b) $6 \sin \theta \cos \theta$

(c) $\frac{1}{2} \sin \frac{\theta}{2} \cos \frac{\theta}{2}$ (d) $\cos^2 \frac{3\theta}{2} - \sin^2 \frac{3\theta}{2}$

(e) $1 - 2 \sin^2 \frac{\theta}{4}$ (f) $2 \cos^2 \left(\frac{7}{2}\theta\right) - 1$

(g) $8 \sin^2 2\theta - 4$ (h) $1 - 2 \sin^2\left(\frac{\pi}{4} - \frac{x}{2}\right)$

3. If $\cos \theta = -\frac{4}{5}, \frac{\pi}{2} \leqslant \theta \leqslant \pi$, find the value of $\sin 2\theta$ and $\cos 2\theta$. Determine the quadrant of angle 2θ.

4. If $\sin \theta = \frac{12}{13}, 0 \leqslant \theta \leqslant \frac{\pi}{2}$, evaluate $\sin 2\theta$ and $\cos 2\theta$. Determine the quadrant of angle 2θ.

5. If $\sin \theta = \frac{2}{3}, 0 \leqslant \theta \leqslant \frac{\pi}{2}$, find the value of $\sin 4\theta$.

6. If $\cos \theta = \frac{2}{5}, \frac{3\pi}{2} \leqslant \theta \leqslant 2\pi$, find the values of $\csc 2\theta$ and $\sec 2\theta$.

7. If $\tan a = \frac{1}{2}, 0 \leqslant a \leqslant \frac{\pi}{2}$, find the value of $\tan 2a$.

8. If $\tan a = 2, -2\pi \leqslant a \leqslant -\frac{3\pi}{2}$, evaluate $\tan 4a$.

9. Develop formulas for

(a) $\sin 3\theta$ in terms of $\sin \theta$. (b) $\cos 3\theta$ in terms of $\cos \theta$.

(c) $\tan 3\theta$ in terms of $\tan \theta$. (d) $\cos 4\theta$ in terms of $\cos \theta$.

10. Find the exact values.

(a) $\sin 67\frac{1}{2}°$ (b) $\cos 112\frac{1}{2}°$ (c) $\tan 22.5°$

(d) $\sin\left(-\frac{\pi}{8}\right)$ (e) $\cos \frac{\pi}{16}$ (f) $\tan 33.75°$

Answers

1. (a) $\cos^2 2x - \sin^2 2x$ or $1 - 2\sin^2 2x$ or $2\cos^2 2x - 1$ (b) $2 \sin \frac{3}{2}x \cos \frac{3}{2}x$

(c) $\dfrac{2 \tan 3x}{1 - \tan^2 3x}$ (d) $2 \sin \frac{1}{4}x \cos \frac{1}{4}x$

(e) $\cos^2 \frac{1}{3}x - \sin^2 \frac{1}{3}x$ (f) $\dfrac{-2 \tan \frac{7}{2}x}{1 - \tan^2 \frac{7}{2}x}$

2. (a) $\sin 6\theta$ (b) $3 \sin 2\theta$ (c) $\frac{1}{4} \sin \theta$ (d) $\cos 3\theta$

(e) $\cos \dfrac{\theta}{2}$ (f) $\cos 7\theta$ (g) $-4 \cos 4\theta$ (h) $\sin x$

3. $\sin 2\theta = -\frac{24}{25}$, $\cos 2\theta = \frac{7}{25}$, 4th quadrant

4. $\sin 2\theta = \frac{120}{169}$, $\cos 2\theta = -\frac{119}{169}$, 2nd quadrant

5. $\dfrac{8\sqrt{5}}{81}$

6. $\csc 2\theta = \dfrac{25}{-4\sqrt{21}}$, $\sec 2\theta = \dfrac{25}{-17}$

7. $\frac{4}{3}$ **8.** $\frac{24}{7}$

9. (a) $3 \sin \theta - 4 \sin^3 \theta$ (b) $4 \cos^3 \theta - 3 \cos \theta$

(c) $\dfrac{3 \tan \theta - \tan^3 \theta}{1 - 3 \tan^2 \theta}$

(d) $8 \cos^4 \theta - 8 \cos^2 \theta + 1$

10. (a) $\sqrt{\dfrac{\sqrt{2} + 1}{2\sqrt{2}}}$ (b) $-\sqrt{\dfrac{\sqrt{2} - 1}{2\sqrt{2}}}$

(c) $-1 + \sqrt{2}$ (d) $-\sqrt{\dfrac{\sqrt{2} - 1}{2\sqrt{2}}}$

(e) $\sqrt{\dfrac{\dfrac{1 + \sqrt{2}}{2\sqrt{2}} + 1}{2}}$ (f) $\dfrac{-1 + \sqrt{4 + 2\sqrt{2}}}{1 + \sqrt{2}}$