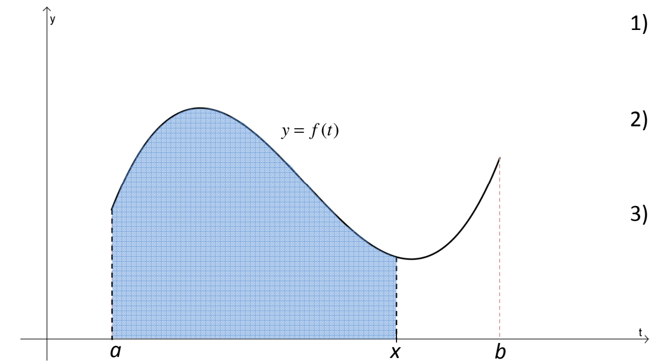
Previewing the Fundamental Theorem of Calculus



1) Consider the graph of the continuous function *f*(*t*) shown on the left.

2) Choose any *x* greater than *a* in the interval [*a*, *b*] and mark it on the *t*-axis.

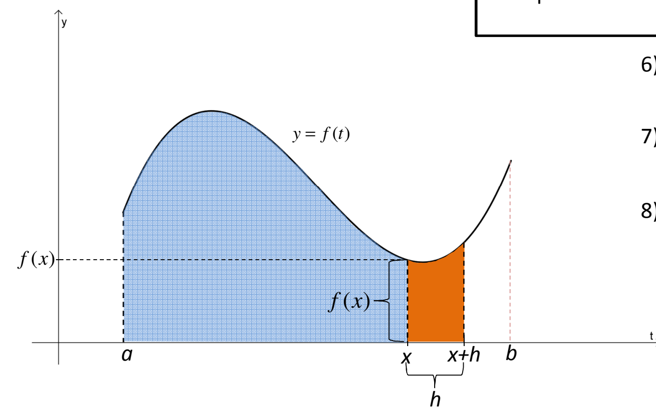
3) Shade in the region between the graph of *f* and the *t*-axis from *a* to *x*.

4) The shaded area represents the definite integral $\int_a^x f(t)dt$.

5) Notice how the integral depends on which *x* we choose. Therefore, the integral is a function of *x* on the interval [*a*, *b*]. Call this function *F*.

So, we have $F(x) = \int_a^x f(t)dt.$

From previous slide: $F(x) = \int_a^x f(t)dt$



6) Draw another shaded region to the right of *x*.

7) Call the width of the new shaded region *h*.

8) The area of the new shaded region is

$$F(x+h) - F(x)$$

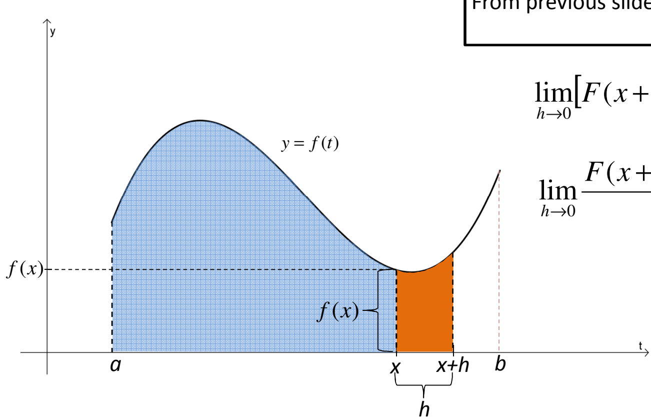
9) For small values of *h*, we see that the area of the new shaded region is approximately equal to the area of the rectangle with base *h* and height $f(x)$.

That is, $F(x+h) - F(x) \approx hf(x)$

10) As *h* approaches zero, $hf(x)$ becomes an increasingly better approximation of the area, **with equality in the limit**.

That is, $\lim_{h\to 0}[F(x+h) - F(x)] = \lim_{h\to 0}[hf(x)]$

From previous slides: $F(x) = \int_a^x f(t)dt$



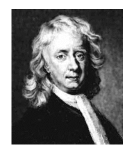
$$\lim_{h\to 0}[F(x+h) - F(x)] = \lim_{h\to 0}[hf(x)]$$

$$\lim_{h\to 0}\frac{F(x+h) - F(x)}{h} = \lim_{h\to 0} f(x)$$

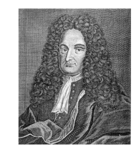
$$\frac{d}{dx}F(x) = f(x)$$

Therefore,

$$\frac{d}{dx}\int_a^x f(t)dt = f(x)$$



$$\frac{d}{dx}\int_a^x f(t)dt = f(x)$$



- This result is the **first part** of the **Fundamental Theorem of Calculus.**

Why is it so important?

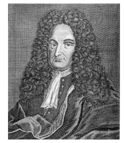
- It connects differential calculus to integral calculus.
- It says that the processes of integration and differentiation are inverses of each other.
- It says that every continuous function is the derivative of some other function.
- It says that every continuous function has an antiderivative.

Example Find $\frac{d}{dx}\int_5^x 2t\cos t\, dt$

$$\frac{d}{dx}\int_5^x 2t\cos t\, dt = 2x\cos x$$

A formal proof of the Fundamental Theorem of Calculus will be studied soon!

$$\frac{d}{dx}\int_a^x f(t)dt = f(x)$$



The first part of the Fundamental Theorem of Calculus (above) means that the integral is an antiderivative of $f(x)$.

So, if $F(x)$ is any antiderivative of $f(x)$, we get

$$\int_a^x f(t)dt = F(x)+C,$$ where *C* is some constant.

Setting *x* equal to *a* gives

$$\int_a^a f(t)dt = F(a)+C$$

$$0 = F(a)+C$$

$$C = -F(a)$$

Therefore, $$\int_a^x f(t)dt = F(x)-F(a)$$

$$\int_a^x f(t)dt = F(x)-F(a)$$

- This result is the **second part** of the **Fundamental Theorem of Calculus.**

Why is it so important?

- It says that integrals can be calculated without calculating limits and Riemann sums!

Example Evaluate $\int_{-2}^{4}\left(x^2+5\right)dx.$

An antiderivative of x^2+5 is $\frac{x^3}{3}+5x.$

Therefore, $$\int_{-2}^{4}\left(x^2+5\right)dx = \left[\frac{4^3}{3}+5(4)\right]-\left[\frac{(-2)^3}{3}+5(-2)\right]$$

$$= 54$$

A formal proof of the Fundamental Theorem of Calculus will be studied soon!