TOUGH DERIVATIVES
ASSIGNED WORK (SOLUTIONS)

1. **(a)**

$$f(x) = (2x-5)^3(3x^2+4)^5$$

$$f'(x) = (2x-5)^3(5)(3x^2+4)^4(6x) + (3x^2+4)^5(3)(2x-5)^2(2)$$

$$= 30x(2x-5)^3(3x^2+4)^4 + 6(3x^2+4)^5(2x-5)^2$$

$$= 6(2x-5)^2(3x^2+4)^4[5x(2x-5)+(3x^2+4)]$$

$$= 6(2x-5)^2(3x^2+4)^4(10x^2-25x+3x^2+4)$$

$$= 6(2x-5)^2(3x^2+4)^4(13x^2-25x+4)$$

(b)

$$g(x) = (8x^3)(4x^2+2x-3)^5$$

$$g'(x) = (8x^3)(5)(4x^2+2x-3)^4(8x+2) + (4x^2+2x-3)^5(24x^2)$$

$$= 40x^3(4x^2+2x-3)^4(8x+2) + 24x^2(4x^2+2x-3)^5$$

$$= 8x^2(4x^2+2x-3)^4[5x(8x+2)+3(4x^2+2x-3)]$$

$$= 8x^2(4x^2+2x-3)^4(40x^2+10x+12x^2+6x-9)$$

$$= 8x^2(4x^2+2x-3)^4(52x^2+16x-9)$$

(c)

$$y = (5+x)^2(4-7x^3)^6$$

$$y' = (5+x)^2(6)(4-7x^3)^5(-21x^2) + (4-7x^3)^6(2)(5+x)$$

$$= -126x^2(5+x)^2(4-7x^3)^5 + 2(5+x)(4-7x^3)^6$$

$$= 2(5+x)(4-7x^3)^5[-63x^2(5+x)+4-7x^3]$$

$$= 2(5+x)(4-7x^3)^5(4-315x^2-70x^3)$$

(d)

$$h(x) = \frac{6x-1}{(3x+5)^4}$$

$$h'(x) = \frac{(3x+5)^4(6)-(6x-1)(4)(3x+5)^3(3)}{\left((3x+5)^4\right)^2}$$

$$= \frac{6(3x+5)^3[(3x+5)-2(6x-1)]}{(3x+5)^8}$$

$$= \frac{6(-9x+7)}{(3x+5)^5}$$

(e)

$$y=\frac{\left(2x^2-5\right)^3}{(x+8)^2}$$

$$\frac{dy}{dx}=\frac{(x+8)^2(3)\left(2x^2-5\right)^2(4x)-\left(2x^2-5\right)^3(2)(x+8)}{\left((x+8)^2\right)^2}$$

$$=\frac{2(x+8)\left(2x^2-5\right)^2\left[6x(x+8)-\left(2x^2-5\right)\right]}{(x+8)^4}$$

$$=\frac{2\left(2x^2-5\right)^2\left(4x^2+48x+5\right)}{(x+8)^3}$$

(f)

$$f(x)=\frac{-3x^4}{\sqrt{4x-8}}$$

$$=\frac{-3x^4}{(4x-8)^{\frac{1}{2}}}$$

$$f'(x)=\frac{(4x-8)^{\frac{1}{2}}\left(-12x^3\right)-\left(-3x^4\right)\left(\frac{1}{2}\right)(4x-8)^{-\frac{1}{2}}(4)}{\left((4x-8)^{\frac{1}{2}}\right)^2}$$

$$=\frac{-6x^3(4x-8)^{-\frac{1}{2}}\left[2(4x-8)-x\right]}{4x-8}$$

$$=\frac{-6x^3(7x-16)}{(4x-8)^{\frac{3}{2}}}$$

$$=\frac{-3x^3(7x-16)}{(4x-8)^{\frac{3}{2}}}$$

(g)

$$g(x)=\left(\frac{2x+5}{6-x^2}\right)^4$$

$$g'(x)=4\left(\frac{2x+5}{6-x^2}\right)^3\left(\frac{\left(6-x^2\right)(2)-(2x+5)(-2x)}{\left(6-x^2\right)^2}\right)$$

$$=4\left(\frac{2x+5}{6-x^2}\right)^3\left(\frac{2\left(6+x^2+5x\right)}{\left(6-x^2\right)^2}\right)$$

$$=8\left(\frac{2x+5}{6-x^2}\right)^3\left(\frac{(x+2)(x+3)}{\left(6-x^2\right)^2}\right)$$

(h) $y=\left[\dfrac{1}{\left(4x+x^2\right)^3}\right]^3$

$$=(4x+x^2)^{-9}$$

$$\frac{dy}{dx}=-9(4x+x^2)^{-10}(4+2x)$$

(i)

$$h(x)=\frac{-4\sqrt{3x+2}}{\left(2x-x^3\right)^2}$$

$$=\frac{-4(3x+2)^{\frac{1}{2}}}{\left(2x-x^3\right)^2}$$

$$h'(x)=-4\,\frac{\left(2x-x^3\right)^2\frac{3}{2}(3x+2)^{-\frac{1}{2}}-2(3x+2)^{\frac{1}{2}}\left(2x-x^3\right)\left(2-3x^2\right)}{\left(\left(2x-x^3\right)^2\right)^2}$$

$$=\frac{2\left(2x-x^3\right)(3x+2)^{-\frac{1}{2}}\left[-3\left(2x-x^3\right)+4(3x+2)\left(2-3x^2\right)\right]}{\left(2x-x^3\right)^4}$$

$$=\frac{2(3x+2)^{-\frac{1}{2}}\left(-6x+3x^3+24x-36x^3+16-24x^2\right)}{\left(2x-x^3\right)^3}$$

$$=\frac{2(3x+2)^{-\frac{1}{2}}\left(-33x^3-24x^2+18x+16\right)}{\left(2x-x^3\right)^3}$$

(j)

$$y=\sqrt{\frac{x^2+1}{x^2-1}}=\frac{\left(x^2+1\right)^{\frac{1}{2}}}{\left(x^2-1\right)^{\frac{1}{2}}}$$

$$\frac{dy}{dx}=\frac{\left(x^2-1\right)^{\frac{1}{2}}\frac{1}{2}\left(x^2+1\right)^{-\frac{1}{2}}(2x)-\left(x^2+1\right)^{\frac{1}{2}}\frac{1}{2}\left(x^2-1\right)^{-\frac{1}{2}}(2x)}{\left(\left(x^2-1\right)^{\frac{1}{2}}\right)^2}$$

$$=\frac{x\left(x^2+1\right)^{-\frac{1}{2}}\left(x^2-1\right)^{-\frac{1}{2}}\left[\left(x^2-1\right)-\left(x^2+1\right)\right]}{x^2-1}$$

$$=\frac{x(-2)}{\left(x^2+1\right)^{\frac{1}{2}}\left(x^2-1\right)^{\frac{3}{2}}}$$

$$=\frac{-2x}{\left(x^2+1\right)^{\frac{1}{2}}\left(x^2-1\right)^{\frac{3}{2}}}$$

(k) $f(x)=[2x+(3x^2-5x)^3]^5$

$f'(x)=5[2x+(3x^2-5x)^3]^4[2+3(3x^2-5x)^2(6x-5)]$

(l)

$$g(x)=\sqrt{2x+\sqrt{x^3}}\quad g'(x)=\frac{1}{2}\left(2x+x^{\frac{3}{2}}\right)^{-\frac{1}{2}}\left(2+\frac{3}{2}x^{\frac{1}{2}}\right)$$

$$=\left(2x+x^{\frac{3}{2}}\right)^{\frac{1}{2}}\qquad=\frac{2+\frac{3}{2}\sqrt{x}}{2\sqrt{2x+\sqrt{x^3}}}$$

2. (a)

$$y=\frac{\sqrt{x}}{\sqrt{x}+1}$$

$$\frac{dy}{dx}=\frac{\left(\sqrt{x}+1\right)\left(\frac{1}{2}x^{-\frac{1}{2}}\right)-\sqrt{x}\left(\frac{1}{2}x^{-\frac{1}{2}}\right)}{\left(\sqrt{x}+1\right)^2}$$

$$=\frac{\frac{1}{2}x^{-\frac{1}{2}}\left(\sqrt{x}+1-\sqrt{x}\right)}{\left(\sqrt{x}+1\right)^2}$$

$$=\frac{\frac{1}{2}x^{-\frac{1}{2}}}{\left(\sqrt{x}+1\right)^2}$$

$$=\frac{1}{2\sqrt{x}\left(\sqrt{x}+1\right)^2}$$

(b)

$$y=\frac{\sqrt{2x+1}}{\sqrt{x+3}}$$

$$=\sqrt{\frac{2x+1}{x+3}}$$

$$\frac{dy}{dx}=\frac{1}{2}\left(\frac{2x+1}{x+3}\right)^{-\frac{1}{2}}\left(\frac{2(x+3)-(2x+1)}{(x+3)^2}\right)$$

$$=\left(\frac{5}{2(x+3)^2}\right)\left(\sqrt{\frac{x+3}{2x+1}}\right)$$

$$=\frac{5}{2(2x+1)^{\frac{1}{2}}(x+3)^{\frac{3}{2}}}$$

3. $y = 3\{x-[x-3(x+2)^2]^{-1}\}$

$$\frac{dy}{dx} = 3\{1-(-1)[x-3(x+2)^2]^{-2}[1-6(x+2)]\}$$

$$= 3\{1+[x-3(x+2)^2]^{-2}[1-6x-12]\}$$

$$= 3\{1+[x-3(x+2)^2]^{-2}[-(6x+11)]\}$$

$$= 3\{1-[6x+11][x-3(x+2)^2]^{-2}\}$$

$$= 3\left\{1-\frac{6x+11}{\left[x-3(x+2)^2\right]^2}\right\}$$

4. $s = 3-2t-[t^{-2}-(3t+5)^4]^5$

$$\frac{ds}{dt} = -2-5[t^{-2}-(3t+5)^4]^4[-2t^{-3}-4(3t+5)^3(3)]$$

$$= -2-5[t^{-2}-(3t+5)^4]^4[-2t^{-3}-12(3t+5)^3]$$

$$= -2+10[t^{-2}-(3t+5)^4]^4[t^{-3}+6(3t+5)^3]$$

5.

$$f(t)=\left(\frac{\sqrt[3]{1-2t}}{1+t^2}\right)^2$$

$$f'(t)=2\left(\frac{(1-2t)^{\frac{1}{3}}}{1+t^2}\right)$$

$$\times\left(\frac{(1+t^2)\left(\frac{1}{3}(1-2t)^{-\frac{2}{3}}(-2)\right)-(1-2t)^{\frac{1}{3}}(2t)}{\left(1+t^2\right)^2}\right)$$

$$\therefore f'(0)=2(1)\left(\frac{-\frac{2}{3}-0}{1}\right)$$

$$=-\frac{4}{3}$$

6. Let $y = -2(x+[2x-5(x-2)^3]^{-1})$

$$\frac{dy}{dx} = -2(1-[2x-5(x-2)^3]^{-2}[2-15(x-2)^2])$$

7.

$$g(x)=\sqrt{[f(x)]^2-1}\ ,\ f(x)=3x-1$$

$$=\left[(f(x))^2-1\right]^{\frac{1}{2}}$$

8. $y = 4x^2(3x^2-5x)^3$

$$y' = 4x^2(3)(3x^2-5x)^2(6x-5)+(3x^2-5x)^3(8x)$$

$$= 12x^2(6x-5)(3x^2-5x)^2+8x(3x^2-5x)^3$$

when $x = 2$

$$y' = 12(2)^2(12-5)(12-10)^2+16(12-10)^3$$

$$= (48)(7)(4)+16(8)$$

$$= 1344+128$$

$$= 1472$$

Equation of the tangent at the point (2, 128)

$$y-128 = 1472(x-2)$$

$$y-128 = 1472x-2944$$

$$y = 1472x-2816$$