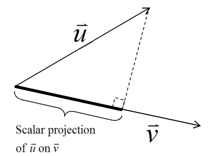
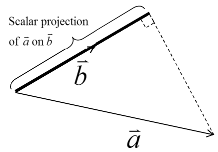
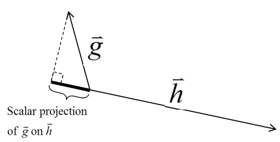
Scalar Projection

The following diagrams illustrate the concept of the scalar projection of one vector onto another:



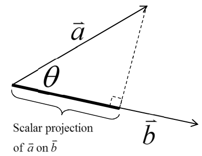


What if the angle between the two vectors is obtuse?



Calculating the scalar projection.

To calculate the scalar projection of one vector onto another, consider the following diagram:



$$\cos\theta = \frac{\text{scalar projection of } \vec{a} \text{ on } \vec{b}}{|\vec{a}|}$$

$$\boxed{\text{scalar projection of } \vec{a} \text{ on } \vec{b} = |\vec{a}|\cos\theta}$$

Notes about the scalar projection:

- the scalar projection is a scalar value
- if the angle between the vectors is acute, the scalar projection is positive
- if the angle between the vectors is obtuse, the scalar projection is negative
- if the angle between the vectors is 90°, the scalar projection is 0

More on scalar projection…

$$\vec{a} \bullet \vec{b} = |\vec{a}||\vec{b}|\cos\theta$$

$$\cos\theta = \frac{\vec{a} \bullet \vec{b}}{|\vec{a}||\vec{b}|}$$

$$\text{scalar projection of } \vec{a} \text{ on } \vec{b} = |\vec{a}|\cos\theta$$

$$\text{scalar projection of } \vec{a} \text{ on } \vec{b} = |\vec{a}|\left(\frac{\vec{a} \bullet \vec{b}}{|\vec{a}||\vec{b}|}\right)$$

$$\boxed{\text{scalar projection of } \vec{a} \text{ on } \vec{b} = \frac{\vec{a} \bullet \vec{b}}{|\vec{b}|}}$$

Note: the scalar projection of $\vec{a}$ on $\vec{b}$ can be written as $|\vec{a} \downarrow \vec{b}|$.

Question for Discussion: Are $|\vec{a} \downarrow \vec{b}|$ and $|\vec{b} \downarrow \vec{a}|$ equal?

Some examples…

1) If $|\vec{u}| = 12$, $|\vec{v}| = 7$ and the angle between $\vec{u}$ and $\vec{v}$ is $54°$, determine the scalar projection of $\vec{u}$ on $\vec{v}$ and the scalar projection of $\vec{v}$ on $\vec{u}$.

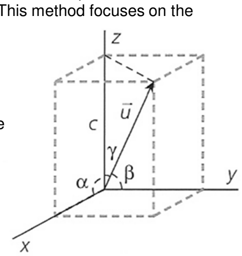
2) If $\vec{a} = (5, 3, -2)$ and $\vec{b} = (-1, 6, 7)$, determine the scalar projection of $\vec{a}$ on $\vec{b}$.

Direction Cosines and Direction Angles

Now that we have discussed scalar projection, we can develop an alternative technique for describing the direction of a vector. This method focuses on the **direction angles** of the vector.

Direction Angles

The **direction angles** of a vector (*a*, *b*, *c*) are the angles *α*, *β* and *γ* that the vector makes with the positive *x*-, *y*- and *z*- axes, respectively, where $0° \le \alpha, \beta, \gamma \le 180°$.



The **direction cosines** of a vector are simply the cosines of the direction angles *α*, *β* and *γ*. The direction cosines can be found using the idea of scalar projection. For a vector $\vec{u} = (a, b, c)$ the direction cosines are

$$\cos\alpha = \frac{a}{|\vec{u}|} \qquad \cos\beta = \frac{b}{|\vec{u}|} \qquad \cos\gamma = \frac{c}{|\vec{u}|}$$

Notice that $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

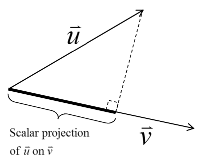
Why do you think this property is true?

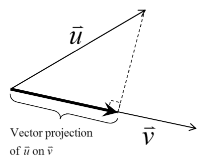
Example

Determine the direction cosines and direction angles for the vector $\left(-2\sqrt{2}, 4, -5\right)$.

Vector Projection

So far, we have only considered the scalar projection of a vector on another vector. To determine the **vector projection** of a vector on another vector, we simply multiply the scalar projection by a unit vector in the same direction as the vector onto which we are projecting.





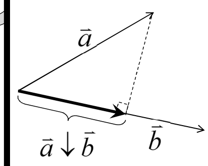
<u>Notes on the Vector Projection</u>

- the vector projection is a vector
- the vector projection acts in the same or opposite direction as the vector onto which the projection occurs (depending on the angle between the two vectors)

The vector projection of $\vec{a}$ on $\vec{b}$ can be written as $\vec{a} \downarrow \vec{b}$.

Calculating a Vector Projection

To calculate a vector projection, we simply multiply the corresponding scalar projection by a unit vector in the same direction as the vector onto which we are projecting.



$$\vec{a} \downarrow \vec{b} = \frac{\vec{a} \bullet \vec{b}}{|\vec{b}|}\left(\frac{\vec{b}}{|\vec{b}|}\right)$$

$$\vec{a} \downarrow \vec{b} = \frac{\vec{a} \bullet \vec{b}}{|\vec{b}|^2}\vec{b}$$

$$\boxed{\vec{a} \downarrow \vec{b} = \frac{\vec{a} \bullet \vec{b}}{\vec{b} \bullet \vec{b}}\vec{b}}$$

Example

Find the vector projection of $\vec{a} = (4, 5)$ on $\vec{b} = (3, -1)$.