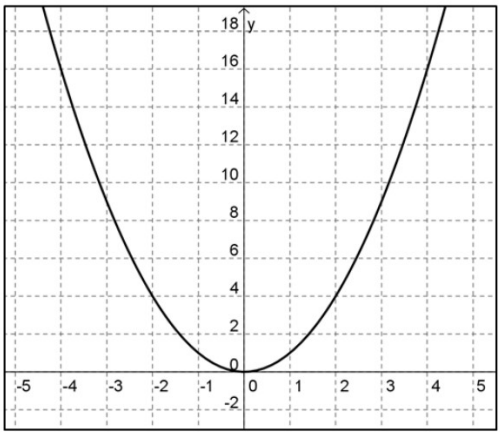
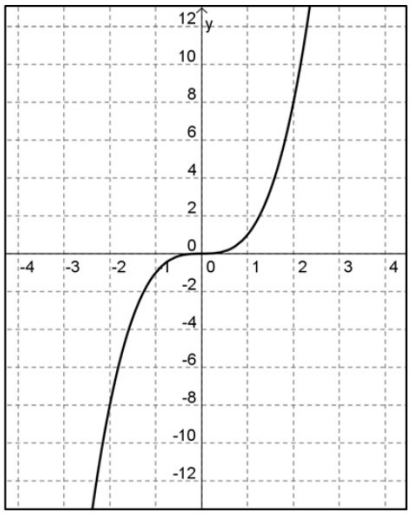
One-to-One Functions

Recall:

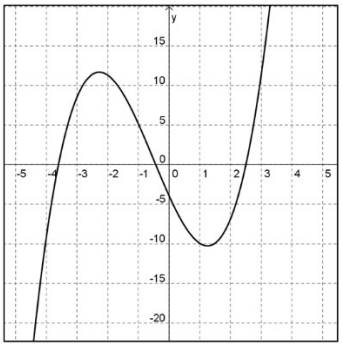
A **function** is a relation for which each element of the domain corresponds to exactly one element of the range. That is, for each *x*, there is exactly one *y*. These relations pass the vertical line test).

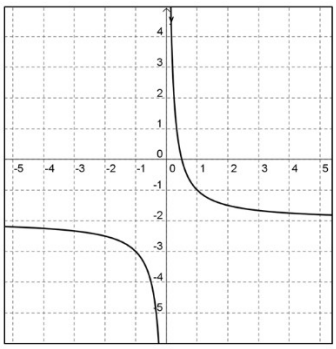


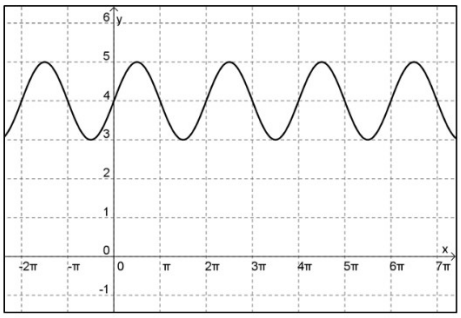
Furthermore, a function is **one-to-one** if we also have that each element of the range corresponds to exactly one element of the domain. That is, for each *y*, there is exactly one *x*. These relations pass the vertical line test and the horizontal line test.



Circle all of the **one-to-one functions**:







$$y = |x + 3|$$

x	y
1	10
2	20
3	30
4	20
5	10

x	y
1	90
2	80
3	70
2	60
1	50

$$f(x) = 4\sqrt{x+6} - 7$$

$$y = \frac{1}{x^2 + 5}$$

$$y = \pm\sqrt{x}$$

$$y = -2(5)^x + 7$$

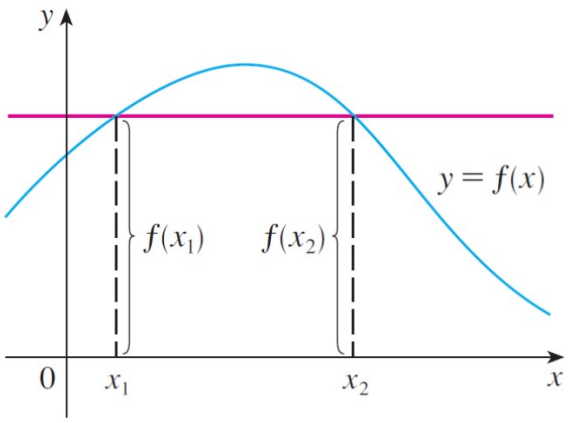
Getting Formal

DEFINITION One-to-One Function

A function $f(x)$ is **one-to-one** on a domain D if $f(a) \neq f(b)$ whenever $a \neq b$.

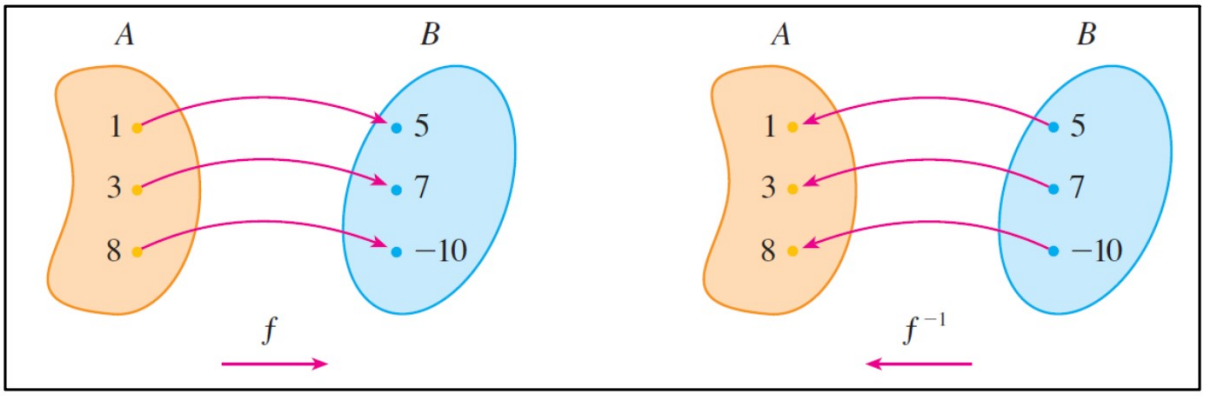
In other words, a function is one-to-one if it never takes on the same value twice.

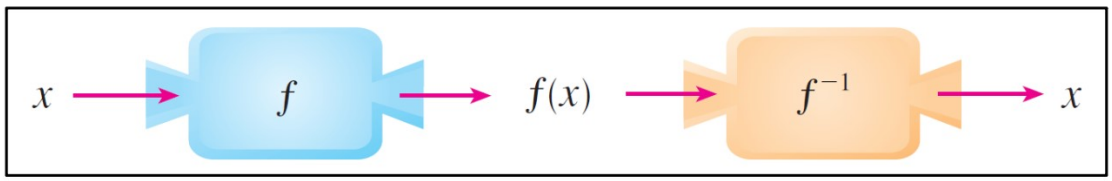
The function shown on the right is not a one-to-one function.



Inverses

Recall that the inverse of a function "undoes" or "reverses" the effect of the function.





$$f^{-1}(x) = y \iff f(y) = x$$

$$f^{-1}(f(x)) = x$$

$$f(f^{-1}(x)) = x$$

Determine the equation for the inverse of the following function.

$f(x) = 2(x-5)^2 - 6$

Graphing the Inverse Parametrically

The use of parametric equations allows us to easily graph a function and its inverse using technology.

Graphing $y = f(x)$ and $y = f^{-1}(x)$ Parametrically

We can graph any function $y = f(x)$ as

$$x_1 = t, \quad y_1 = f(t).$$

Interchanging t and $f(t)$ produces parametric equations for the inverse:

$$x_2 = f(t), \quad y_2 = t.$$

Consider the function $f(x) = x^3 - 4x^2 + 5x - 7$.

1) Express $f(x)$ using parametric equations.

2) Express $f^{-1}(x)$ using parametric equations.

3) Use a calculator to graph $f(x)$, $f^{-1}(x)$ and the line $y = x$.

Logarithms

1) Evaluate the following logarithms.

a) $\log_2 32 =$ b) $\log_5 \frac{1}{125} =$ c) $\log_4 \sqrt[3]{16} =$

2) Complete the following logarithm laws.

$\log_a xy =$

$\log_a \frac{x}{y} =$

$\log_a x^y =$

$\log_a a^x =$ $a^{\log_a x} =$ $\log_a x =$ ———

Special Logarithms

Common Logarithm	Natural Logarithm
$\log_{10} x =$	$\log_e x =$

Examples

1) Solve the equation $2(1.75)^x = 16$

2) Solve the equation $6(e)^{x+4} - 81 = 130$

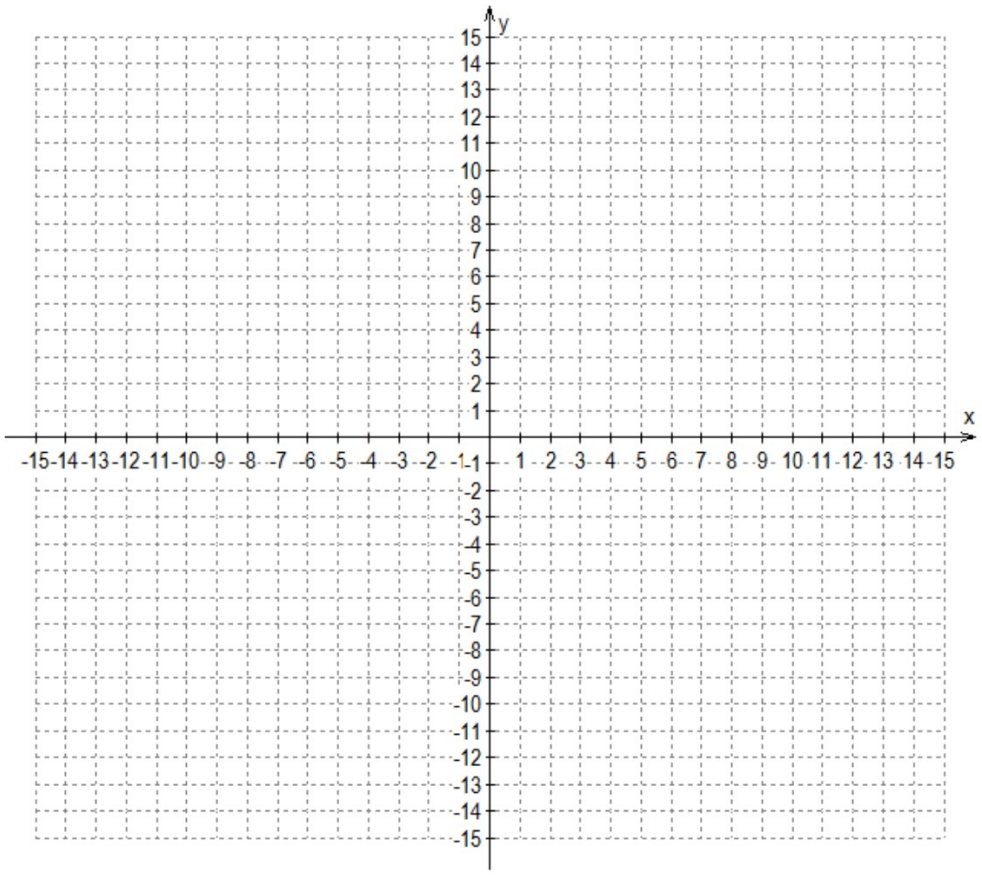
3) Solve the following equation for *y*.

$$\ln(y+8)-2x=3$$

4) Solve the following equation.

$$e^{x}-2e^{-x}=-1$$

5) Sketch the graph of the function $y = -2\log_3(x+4)$.



6) Evan invests $1000 in an account that earns 5.25% compounded annually. How long will it take for the account to reach $2500?