

EQUIVALENT TRIGONOMETRIC EXPRESSIONS

Setting the Stage

Express each of the following in terms of the related acute angle (you do not need to evaluate).
A sketch will help.

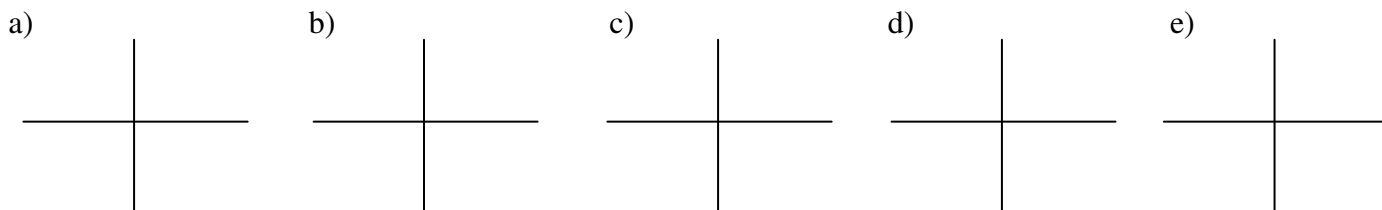
a) $\tan 220^\circ$ b) $\cos 150^\circ$ c) $\sin(-200^\circ)$ d) $\sin \frac{2\pi}{3}$ e) $\sec\left(\frac{5\pi}{4}\right)$

Making Generalizations

Now, for an acute angle x , in radians, consider the following angles:

a) $\pi - x$ b) $\pi + x$ c) $2\pi - x$ d) $2\pi + x$ e) $-x$

Draw each of the above angles in standard position.



What do the five given angles all have in common?....

They all have the same related acute angle of x !

Therefore, the sine, cosine, and tangent of the above five angles are equal to the sine, cosine and tangent of angle x (respectively), with the sign determined by the CAST rule.

Some examples

a) $\cos(\pi + x) =$ b) $\tan(2\pi - x) =$ c) $\sin(\pi - x) =$

The following are very important and should be memorized.

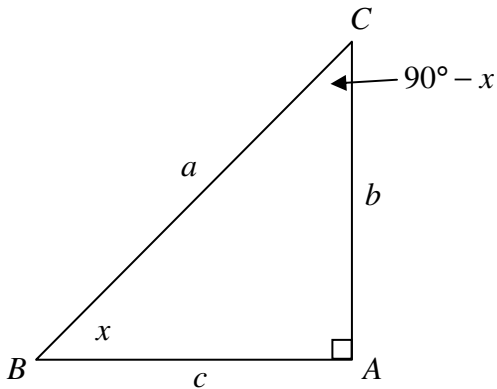
$$\sin(-x) =$$

$$\cos(-x) =$$

$$\tan(-x) =$$

One more thing...

Consider the following right triangle, in which $\angle A = 90^\circ$ and $\angle B = x$. Therefore, we see that $\angle C$ must equal $90^\circ - x$. This diagram leads to the following results:



$$\sin(90^\circ - x) = \cos x$$

$$\cos(90^\circ - x) = \sin x$$

$$\tan(90^\circ - x) = \cot x$$

or, using radians...

$$\sin\left(\frac{\pi}{2} - x\right) = \cos x$$

$$\cos\left(\frac{\pi}{2} - x\right) = \sin x$$

$$\tan\left(\frac{\pi}{2} - x\right) = \cot x$$

The above relationships are also very important and should be memorized!

Let's take a look at how we can use these relationships.

Example

Express each of the following as a trigonometric function of the acute angle x .

a) $\tan\left(\frac{\pi}{2} + x\right)$

b) $\sin\left(\frac{3\pi}{2} - x\right)$

c) $\cos\left(\frac{3\pi}{2} + x\right)$



Note: Throughout this lesson, we have been assuming that the angle x is acute. What if x is not acute? Well, using our knowledge of transformations, a quick inspection of the graphs of $y = \sin x$, $y = \cos x$, $y = \tan x$ and $y = \cot x$ will show that these relationships hold for all values of x . Check it out...now!