# SOLUTIONS

## MHF4U1 - UNIT 3 – POLYNOMIAL FUNCTIONS
### SUPPLEMENTARY REVIEW PROBLEMS

**Multiple Choice**
*In the blank space provided, identify the letter of the choice that best completes the statement or answers the question.*

__c__ 1) The degree of $f(x) = 2x^2 + 6x^3 - x^4$ is

a. 1
b. 2
c. 4
d. 9

__d__ 2) Which of the following statements are true for $f(x) = -2(x-1)(x-2)(x+3)(x+4)$?

i) as $x \to \infty, f(x) \to -\infty$
ii) as $x \to \infty, f(x) \to \infty$
iii) as $x \to -\infty, f(x) \to \infty$
iv) as $x \to -\infty, f(x) \to -\infty$

a. i)
b. i) and ii)
c. i) and iii)
d. i) and iv)

__d__ 3) The function $f(x) = -2x(x-1)(x-2)(x+2)$

a. Has 4 zeros
b. Has an absolute maximum
c. as $x \to -\infty,\ f(x) \to -\infty$
d. all of the above

__c__ 4) A factor of $x^4 - 5x^2 + 4$ is

a. $x - 2$
b. $x - 1$
c. a and b
d. neither a nor b

**Full Solution**

Provide full solutions to the following problems on a separate page.

5) Divide $(x^3 - 7x - 6) \div (x - 3)$.

$$\begin{array}{r|rrrr} 3 & 1 & 0 & -7 & -6 \\ & & 3 & 9 & 6 \\ \hline & 1 & 3 & 2 & 0 \end{array}$$

$\therefore (x^3 - 7x - 6) \div (x - 3) = x^2 + 3x + 2$

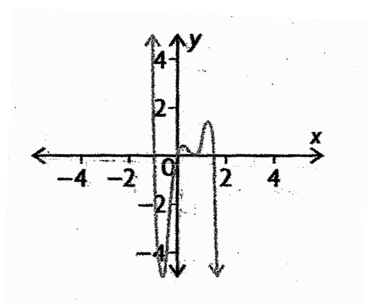
6) Divide $(x^4 - 8x^3 + 2x^2 + 24x + 9) \div (x^2 - 2x - 1)$.

$$
\begin{array}{r}
x^2 - 6x - 9 \\
x^2 - 2x - 1 \overline{\big) x^4 - 8x^3 + 2x^2 + 24x + 9} \\
\underline{x^4 - 2x^3 - x^2 \qquad\qquad} \\
-6x^3 + 3x^2 + 24x \quad \\
\underline{-6x^3 + 12x^2 + 6x \quad} \\
-9x^2 + 18x + 9 \\
\underline{-9x^2 + 18x + 9} \\
0
\end{array}
$$

$$\therefore (x^4 - 8x^3 + 2x^2 + 24x + 9) \div (x^2 - 2x - 1) = x^2 - 6x - 9$$

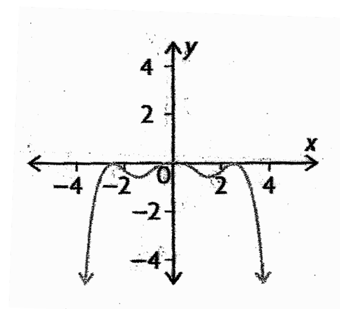
7) For the graphs of polynomial functions shown below, state whether the leading coefficient of the polynomial is positive or negative, and whether its degree is even or odd.

a)




- leading coefficient is negative
- degree is odd

b)




- leading coefficient is negative
- degree is even

8) Factor $x^3 - 5x^2 - x + 5$.

$x^3 - 5x^2 - x + 5$

$= (x-1)(x^2 - 4x - 5)$

$= (x-1)(x+1)(x-5)$

Let $f(x) = x^3 - 5x^2 - x + 5$

$f(1) = 0$ $\therefore (x-1)$ is a factor

| | | | | |
|---|---|---|---|---|
| 1 | 1 | -5 | -1 | 5 |
| | | 1 | -4 | -5 |
| | 1 | -4 | -5 | 0 |

9) Factor $64x^3 - 27$.

$$64x^3 - 27 = (4x)^3 - 3^3$$

$$= (4x-3)(16x^2 + 12x + 9)$$

10) Sketch the graph of $f(x) = x^4 - 3x^3 - 9x^2 + 27x$ using the zeros and end behaviour. You may use the following grid for your sketch if you wish, but please show all calculations on a separate page.

$$f(x) = x^4 - 3x^3 - 9x^2 + 27x$$
$$= x(x^3 - 3x^2 - 9x + 27)$$
$$= x(x-3)(x^2 - 9)$$
$$= x(x-3)(x-3)(x+3)$$
$$= x(x-3)^2(x+3)$$

Let $g(x) = x^3 - 3x^2 - 9x + 27$

$g(3) = 0$ $\quad\therefore (x-3)$ is a factor

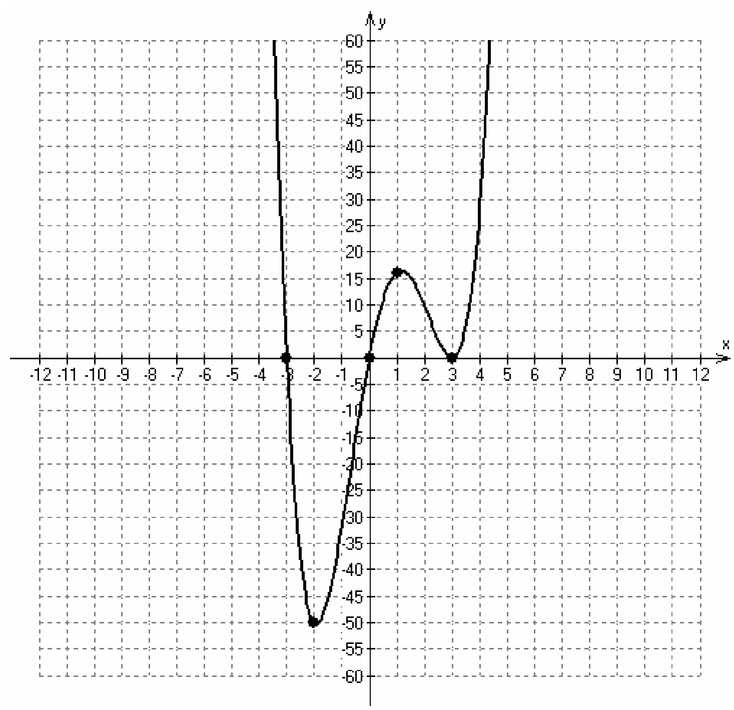
| | | | | |
|---|---|---|---|---|
| 3 | 1 | -3 | -9 | 27 |
| | | 3 | 0 | -27 |
| | 1 | 0 | -9 | 0 |

Zeros are 0, 3 and -3 and there is also a turning point at $x = 3$.

As $x \to \infty,\ f(x) \to \infty$

As $x \to -\infty,\ f(x) \to \infty$

Additional points: $f(-2) = -50$, $f(1) = 16$




11) Given the base function $y = x^3$, write the equation after the graph has been vertically reflected (in the $x$-axis), vertically stretched by a factor of 2, horizontally stretched by a factor of 3, horizontally shifted 4 units right and a vertically shifted 2 units down. You do not need to simplify your answer.

$$y = -2\left(\frac{1}{3}(x-4)\right)^3 - 2$$

12) $2x + 3$ is a factor of $2x^3 + 5x^2 + 5x + p$. Calculate $p$.

Let $f(x) = 2x^3 + 5x^2 + 5x + p$

Since $2x - 3$ is a factor, we know that $f\left(-\frac{3}{2}\right) = 0$

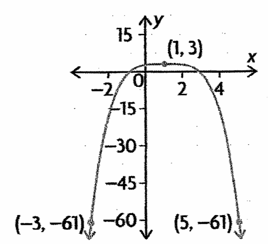
$$\begin{aligned} f\left(-\frac{3}{2}\right) &= 0 \\ f(-1.5) &= 0 \\ 2(-1.5)^3 + 5(-1.5)^2 + 5(-1.5) + p &= 0 \\ -3 + p &= 0 \\ p &= 3 \end{aligned}$$

13) Determine the equation (in factored form) of the cubic function with zeros 2, 4, and -5, and a $y$-intercept of 20.

$$\begin{aligned} y &= a(x-2)(x-4)(x+5) \\ 20 &= a(0-2)(0-4)(0+5) \\ 20 &= a(-2)(-4)(5) \\ 20 &= a(40) \\ a &= \frac{1}{2} \end{aligned}$$

$\therefore$ the equation is $y = \frac{1}{2}(x-2)(x-4)(x+5)$

14) Find the equation for the quartic function whose graph is shown on the right. You do not need to simplify your answer.




The graph of $y = x^4$ has undergone a vertical reflection (in the $x$-axis), a vertical stretch of factor 64, a horizontal stretch of factor 4, a vertical shift 3 units up and a horizontal shift 1 unit right. Therefore, the equation is

$$y = -64\left(\frac{1}{4}(x-1)\right)^4 + 3$$

15) Briefly describe how the graphs of $f(x) = k(x-s)(x-t)(x-u)$ and $g(x) = k(x-s)(x-t)^2$ differ. You may answer in point form.

- the graph of $f(x)$ has three distinct zeros, whereas the graph of $g(x)$ has two distinct zeros
- the graph of $g(x)$ has a turning point at one of its zeros ($t$), whereas the graph of $f(x)$ does not

16) The graph of the function $f(x) = k(x-a)(x-b)^2(x-c)^3$ has zeros at $a$, $b$, and $c$. Briefly describe the appearance of the graph at each of these zeros.

The graph crosses the zero at $a$ without any notable change in behaviour. There is a turning point at the zero at $b$, and thus the graph appears parabolic near this zero. The graph appears "cubic" at the zero at $c$ (its rate of change decreases as it approaches the zero and then increase as it moves away from the zero).

17) Prove that $(x+a)^5 + (x^2+ax)^5 + \left(1+\frac{a}{x}\right)^5$ has no remainder when divided by $x + a$.

Let $f(x) = (x+a)^5 + (x^2+ax)^5 + \left(1+\frac{a}{x}\right)^5$

When $f(x)$ is divided by $x + a$, the remainder is $f(-a)$.

$$\begin{aligned} f(-a) &= (-a+a)^5 + \left[(-a)^2 + a(-a)\right]^5 + \left(1+\frac{a}{-a}\right)^5 \\ &= (0)^5 + (a^2-a^2)^5 + (1-1)^5 \\ &= 0 + 0^5 + 0^5 \\ &= 0 \end{aligned}$$

$\therefore (x+a)^5 + (x^2+ax)^5 + \left(1+\frac{a}{x}\right)^5$ has no remainder when divided by $x + a$.