DIVIDING POLYNOMIALS

You may have noticed that we have added, subtracted, and multiplied polynomials in the past, but we did not do any dividing. This is because dividing polynomials is not nearly as simple as the other three operations.

In grade 11 we did do some dividing of polynomials, but all we looked at were rigged to "factor and cancel." You might even remember the procedure:

1) Factor the numerator and denominator.
2) State the restrictions.
3) Cancel.

Most times though, life is not so easy. The numerator and denominator will not have factors in common, and dividing polynomials takes on a different look. To be able to handle the procedure, we will need to look back a little.

Long Division

Remember when….

$3\overline{)195}$ $4\overline{)2065}$

Notice that $195 \div 3 = 65$ remainder 0 implies $65 \times 3 = 195$.

And $2065 \div 4 = 516$ remainder 1 implies $4 \times 516 + 1 = 2065$.

So, we have: *divisor* × *quotient* + *remainder* = *dividend*

"So what?", you say.

Consider the following question.

Example:

Find $(x^2+7x-3)\div(x+1)$.

Solution:

Notice that x^2+7x-3 does not even factor, so cancellation is not going to happen. So, we are left with long division.

$$x+1\overline{)x^2+7x-3}$$

Exercises:

Divide:

1) $(x^3-5x+9)\div(x-2)$

2) $(x^4-3x^3+2x^2+5x-2)\div(x-4)$

3) $(5x-2x^3+3+x^4)\div(1+2x+x^2)$

Synthetic Division

There is a short form of this procedure that will make the calculation go a lot faster (although to be honest I sometimes forget how it goes, and I just go back to long division…). The basic idea of the procedure is to not write down all the *x* powers and just work with the coefficients. As well, we reverse the procedure, so that we can use adding instead of subtracting.

Example:

$(4x^3 - 5x^2 + 3x - 7) \div (x - 2)$

Solution:

In the process of synthetic division, we are first concerned with what is called the "*k* value." This will be the multiplier we will use in the procedure. Here, the value is 2, since we are dividing by (*x*-2). (By using 2 instead of -2, we can now add instead of subtract at each step.)

Recipe:

1) Decide on the *k* value.
2) List the coefficients in the dividend.
3) Bring down the first coefficient.
4) "Multiply and add" to get the rest of the coefficients.
5) Write out the results using the variable powers.

Exercises:

1) $(5x^4 + 9x^3 - 2x^2 + 1) \div (x - 3)$

2) $(4x^4 - 2x^3 + 5x^2 + 4x + 2) \div (x + 2)$

3) $(12x^3 + 2x^2 + 11x + 16) \div (3x + 2)$

(Hint: $3x + 2 = 3(x + 2/3)$, so divide by $(x + \frac{2}{3})$, then by 3.)