TRANSFORMATIONS OF TRIGONOMETRIC FUNCTIONS

SUPPLEMENTARY PROBLEMS

1. Determine the vertical translation and the phase shift of each function with respect to $y = \sin x$.

a) $y = \sin x + 3$ **b)** $y = \sin x - 1$

c) $y = \sin (x - 45°)$ **d)** $y = \sin \left(x - \frac{3\pi}{4}\right)$

e) $y = \sin (x - 60°) + 1$

f) $y = \sin \left(x + \frac{\pi}{3}\right) + 4$

g) $y = \sin \left(x + \frac{3\pi}{8}\right) - 0.5$

h) $y = \sin (x - 15°) - 4.5$

2. Determine the vertical translation and the phase shift of each function with respect to $y = \cos x$.

a) $y = \cos x + 6$ **b)** $y = \cos x - 3$

c) $y = \cos \left(x + \frac{\pi}{2}\right)$ **d)** $y = \cos (x + 72°)$

e) $y = \cos (x - 30°) - 2$

f) $y = \cos \left(x + \frac{\pi}{6}\right) + 1.5$

g) $y = \cos (x + 110°) + 25$

h) $y = \cos \left(x - \frac{5\pi}{12}\right) - 3.8$

3. Sketch one cycle of the graph of each of the following. State the amplitude, period, domain, and range of the cycle.

a) $y = 3\sin x + 2$ **b)** $y = 2\cos x - 2$

c) $y = 1.5\sin x - 1$ **d)** $y = \frac{1}{2}\cos x + 1$

4. Sketch one cycle of the graph of each of the following. State the amplitude, period, domain, range, and phase shift of the cycle.

a) $y = 2\sin (x - \pi)$ **b)** $y = \cos \left(x - \frac{\pi}{2}\right)$

c) $y = \frac{1}{2}\sin \left(x + \frac{\pi}{2}\right)$ **d)** $y = 3\cos \left(x + \frac{\pi}{4}\right)$

e) $y = -\cos \left(x + \frac{\pi}{2}\right)$

5. Determine the amplitude, period, vertical translation, and phase shift for each function with respect to $y = \sin x$.

a) $y = 2\sin x - 3$ **b)** $y = 0.5\sin (2x) - 1$

c) $y = 6\sin 3(x - 20°)$

d) $y = -5\sin 2\left(x - \frac{\pi}{6}\right) + 1$

6. Determine the amplitude, period, vertical translation, and phase shift for each function with respect to $y = \cos x$.

a) $y = \cos x + 3$ **b)** $y = \cos 3(x - 90°)$

c) $y = -3\cos 4\left(x - \frac{\pi}{4}\right) + 5$

d) $y = 0.8\cos \frac{2}{3}\left(x - \frac{\pi}{3}\right) - 7$

7. Sketch one cycle of the graph of each of the following. State the amplitude, period, domain, range, and phase shift of the cycle.

a) $y = \sin 2\left(x + \frac{\pi}{4}\right)$

b) $y = 2\cos 2\left(x - \frac{\pi}{4}\right) + 1$

c) $y = 3\sin \frac{1}{2}(x - \pi) - 2$

d) $y = 4\cos \frac{1}{3}(x + 2\pi) - 4$

e) $y = -3\sin 2\left(x - \frac{\pi}{4}\right) + 2$

8. Communication Sketch one cycle of the graph of each of the following. State the amplitude, period, domain, range, and phase shift of the cycle.

a) $y = \sin\left(2x - \frac{\pi}{2}\right)$ **b)** $y = \cos\left(\frac{1}{2}x - \pi\right) - 2$

c) $y = 2\sin(3x - \pi) + 2$

d) $y = -3\cos(2x - 4\pi) - 1$

9. Write an equation for the function with the given characteristics, where T is the type, A is the amplitude, P is the period, V is the vertical shift, and H is the horizontal shift.

	T	A	P	V	H
a)	sine	8	2π	-6	none
b)	cosine	7	π	2	none
c)	sine	1	4π	3	π right
d)	cosine	10	$\frac{\pi}{2}$	none	$\frac{\pi}{2}$ left

10. Sketch the graph of each of the following. State the range.

a) $y = 2\sin x + 2,\ 0 \le x < 3\pi$

b) $y = -\cos 3x - 2,\ -\pi \le x \le \pi$

c) $y = 3\cos\left(x - \frac{\pi}{6}\right),\ -2\pi \le x \le 2\pi$

d) $y = 4\sin 2\left(x + \frac{\pi}{4}\right) - 1,\ -\pi \le x \le \pi$

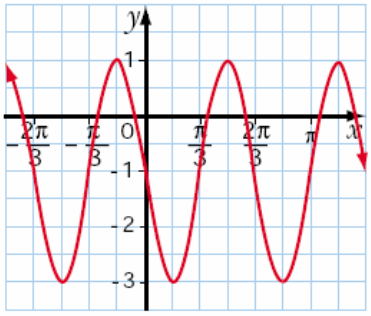
e) $y = -2\sin\left(2x - \frac{\pi}{3}\right) + 1,\ -\pi \le x \le \pi$

f) $y = 5\cos\left(\frac{1}{3}x - \frac{\pi}{3}\right) + 2,\ -\pi \le x \le 3\pi$

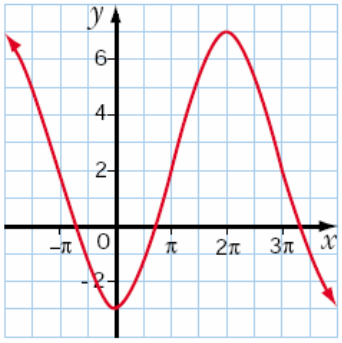
g) $y = 2\sin\left(2x + \frac{\pi}{8}\right),\ -2\pi \le x \le 2\ \pi$

11. Each graph shows part of the sine function of the form $y = a\sin k(x - d) + c$. Determine the values of a, k, d, and c for each graph. Check by graphing.

a)



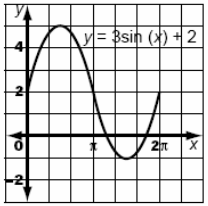
b)



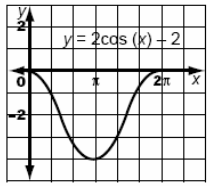
Answers

1. a) vertical translation: 3 units upward; phase shift: none **b)** vertical translation: 1 unit downward; phase shift: none **c)** vertical translation: none; phase shift: $45°$ to the right **d)** vertical translation: none; phase shift: $\frac{3\pi}{4}$ to the right **e)** vertical translation: 1 unit upward; phase shift: $60°$ to the right **f)** vertical translation: 4 units upward; phase shift: $\frac{\pi}{3}$ to the left **g)** vertical translation: 0.5 units downward; phase shift: $\frac{3\pi}{8}$ to the left **h)** vertical translation: 4.5 units downward; phase shift: $15°$ to the right **2. a)** vertical translation: 6 units upward; phase shift: none **b)** vertical translation: 3 units downward; phase shift: none **c)** vertical translation: none; phase shift: $\frac{\pi}{2}$ to the left **d)** vertical translation: none; phase shift: $72°$ to the left **e)** vertical translation: 2 units downward; phase shift: $30°$ to the right **f)** vertical translation: 1.5 units upward; phase shift: $\frac{\pi}{6}$ to the left **g)** vertical translation: 25 units upward; phase shift: $110°$ to the left **h)** vertical translation: 3.8 units downward, phase shift: $\frac{5\pi}{12}$ to the right

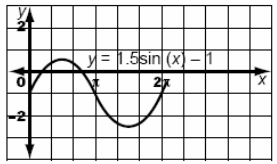
3. a) amplitude: 3, period: 2π, domain: $0 \le x \le 2\pi$, range: $-1 \le y \le 5$



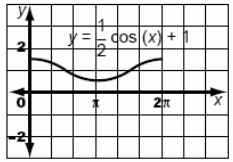
b) amplitude: 2, period: 2π, domain: $0 \le x \le 2\pi$, range: $-4 \le y \le 0$



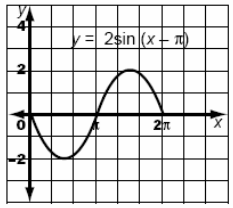
c) amplitude: 1.5, period: 2π, domain: $0 \le x \le 2\pi$, range: $-2.5 \le y \le 0.5$



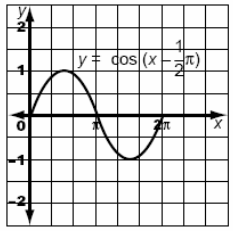
d) amplitude: $\frac{1}{2}$, period: 2π, domain: $0 \le x \le 2\pi$, range: $\frac{1}{2} \le y \le \frac{3}{2}$



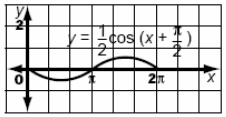
4. a) amplitude: 2, period: 2π, domain: $0 \le x \le 2\pi$, range: $-2 \le y \le 2$, phase shift: π to the right



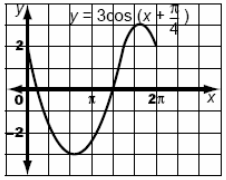
b) amplitude: 1, period: 2π, domain: $0 \le x \le 2\pi$, range: $-1 \le y \le 1$, phase shift: $\frac{\pi}{2}$ to the right



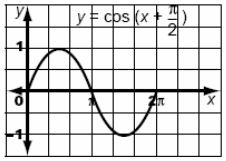
c) amplitude: $\frac{1}{2}$, period: 2π, domain: $0 \le x \le 2\pi$, range: $-\frac{1}{2} \le y \le \frac{1}{2}$, phase shift: $\frac{\pi}{2}$ to the left



d) amplitude: 3, period: 2π, domain: $0 \le x \le 2\pi$, range: $-3 \le y \le 3$, phase shift: $\frac{\pi}{4}$ to the left

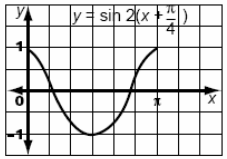


e) amplitude: 1, period: 2π, domain: $0 \le x \le 2\pi$, range: $-1 \le y \le 1$, phase shift: $\frac{\pi}{2}$ to the left

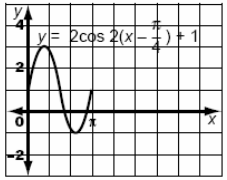


5. a) amplitude: 2, period: 2π, vertical translation: 3 units downward, phase shift: none **b)** amplitude: 0.5, period: π, vertical translation: 1 unit downward, phase shift: none **c)** amplitude: 6, period: $\frac{2\pi}{3}$, vertical translation: none, phase shift: $\frac{\pi}{9}$ to the right **d)** amplitude: 5, period: π, vertical translation: 1 unit upward, phase shift: $\frac{\pi}{6}$ to the right **6. a)** amplitude: 1, period: 2π, vertical translation: 3 units upward, phase shift: none **b)** amplitude: 1, period: $\frac{2\pi}{3}$, vertical translation: none, phase shift: $\frac{\pi}{2}$ to the right **c)** amplitude: 3, period: $\frac{\pi}{2}$, vertical translation: 5 units upward, phase shift: $\frac{\pi}{4}$ to the right **d)** amplitude: 0.8, period: 3π, vertical translation: 7 units downward, phase shift: $\frac{\pi}{3}$ to the right

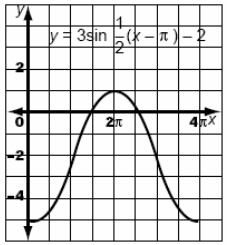
7. a) amplitude: 1, period: π, domain: $0 \le x \le \pi$, range: $-1 \le y \le 1$, phase shift: $\frac{\pi}{4}$ to the left



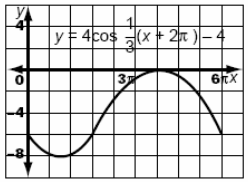
b) amplitude: 2, period: π, domain: $0 \le x \le \pi$, range: $-1 \le y \le 3$, phase shift: $\frac{\pi}{4}$ to the right



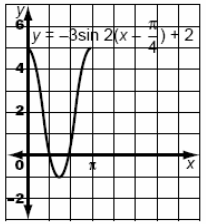
c) amplitude: 3, period: 4π, domain: $0 \le x \le 4\pi$, range: $-5 \le y \le 1$, phase shift: π to the right



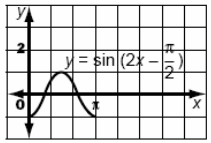
d) amplitude: 4, period: 6π, domain: $0 \le x \le 6\pi$, range: $-8 \le y \le 0$, phase shift: 2π to the left



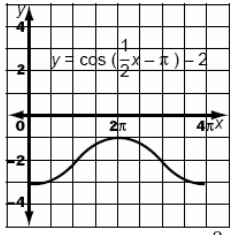
e) amplitude: 3, period: π, domain: $0 \le x \le \pi$, range: $-1 \le y \le 5$, phase shift: $\frac{\pi}{4}$ to the right



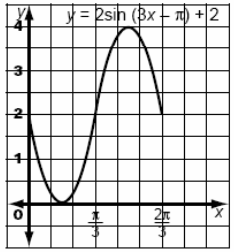
8. a) amplitude: 1, period: π, domain: $0 \le x \le \pi$, range: $-1 \le y \le 1$, phase shift: $\frac{\pi}{4}$ to the right



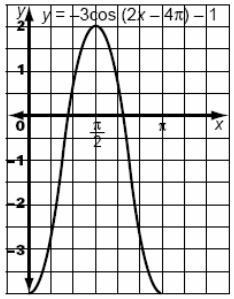
b) amplitude: 1, period: 4π, domain: $0 \le x \le 4\pi$, range: $-3 \le y \le -1$, phase shift: 2π to the right



c) amplitude: 2, period: $\frac{2\pi}{3}$, domain: $0 \le x \le \frac{2\pi}{3}$, range: $0 \le y \le 4$, phase shift: $\frac{\pi}{3}$ to the right

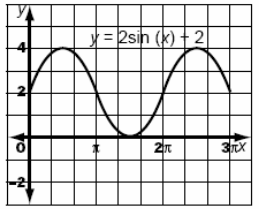


d) amplitude: 3, period: π, domain: $0 \le x \le \pi$, range: $-4 \le y \le 2$, phase shift: 2π to the right

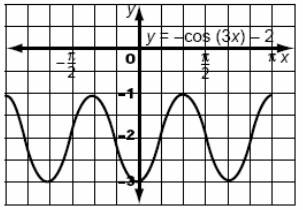


9. a) $y = 8\sin x - 6$ **b)** $y = 7\cos 2x + 2$ **c)** $y = \sin\frac{1}{2}(x - \pi) + 3$ **d)** $y = 10\cos 4\left(x + \frac{\pi}{2}\right)$

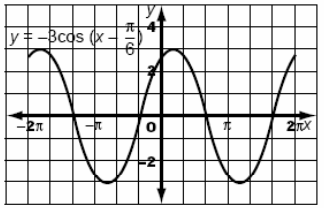
10. a) range: $0 \le y \le 4$



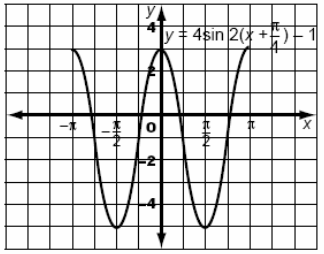
b) range: $-3 \le y \le -1$



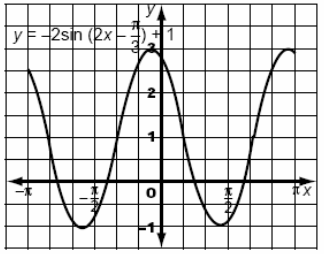
c) range: $-3 \le y \le 3$



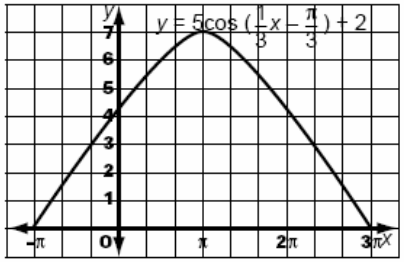
d) range: $-5 \le y \le 3$



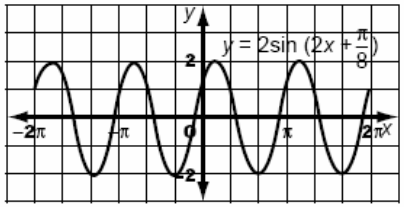
e) range: $-1 \le y \le 3$



f) range: $-0.5 \le y \le 7$



g) range: $-2 \le y \le 2$



11. a) $a = 2,\ k = 3,\ d = \frac{\pi}{3},\ c = -1$ **b)** $a = 5,\ k = \frac{1}{2},\ d = \pi,\ c = 2$