THE FORMAL DEFINITION OF THE LIMIT

Developing the Idea

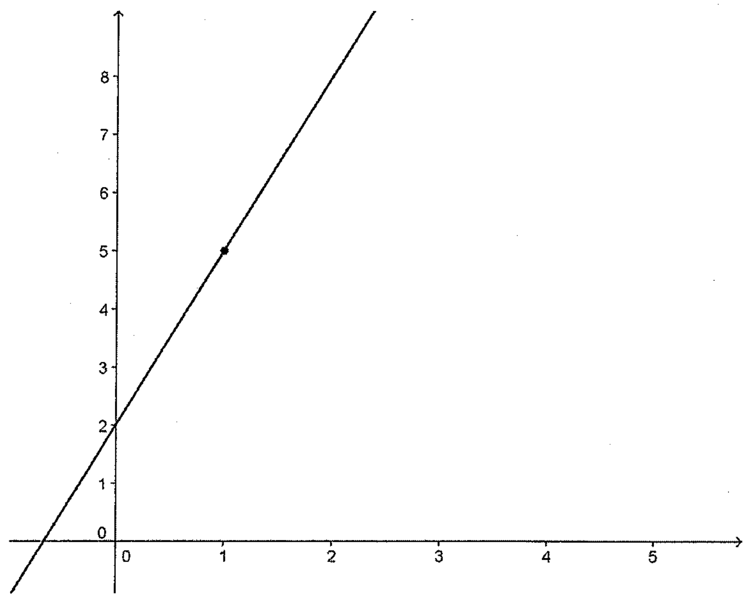
To develop the idea behind the formal definition of the limit, we'll consider the function $f(x) = 3x + 2$.

For $f(x) = 3x + 2$, $\lim_{x \to 1} f(x) =$ 5

In the formal definition of this limit, we consider how close *x* must be to 1 in order to make *f*(*x*) *close* to 5.

Now, the distance from *x* to 1 can be written as $|x-1|$. Similarly, the distance from *f*(*x*) to 5 can be written as $|f(x)-5|$.

Therefore, another way of stating the idea above is to consider how small $|x-1|$ must be in order to make $|f(x)-5|$ *small.*



Now, when we say "small," we aren't being very specific. How small do we need $|f(x)-5|$ to be? That is, how close to 5 do we need *f*(*x*) to be? The answer is, "as close as we want!"

For example, suppose we want *f*(*x*) to be closer to 5 than 0.2.

That is, we want $4.8 < f(x) < 5.2$. Written differently, we need $|f(x)-5| < 0.2$.

What must be true about the value of *x* in order for this requirement to be satisfied?

$|f(x)-5| < 0.2$

$|3x+2-5| < 0.2$

$|3x-3| < 0.2$

$3|x-1| < 0.2$

$\to |x-1| < \frac{0.2}{3}$

$|x-1| < \frac{1}{15}$

$-\frac{1}{15} < x-1 < \frac{1}{15}$

$\to -\frac{1}{15}+1 < x < \frac{1}{15}+1$

$\frac{14}{15} < x < \frac{16}{15}$

We need $\frac{14}{15} < x < \frac{16}{15}$. Written differently, $0 < |x-1| < \frac{1}{15}$.

Question for Discussion

Why do we write $0 < |x-1| < \frac{1}{15}$ instead of simply $|x-1| < \frac{1}{15}$?

$|x-1|$ is the distance from x to 1. This distance should be greater than zero since we are considering values close to 1, but not actually at 1