1) $\lim_{x\to 3} \overbrace{(x+5)}^{f(x)} = 8$

We need to find $\delta>0$ such that if $0<|x-3|<\delta$, then $|f(x)-8|<\varepsilon$ for all $\varepsilon>0$.

$$
\begin{aligned}
\text{Now,} \quad & |f(x)-8| < \varepsilon \\
\Leftrightarrow \quad & |x+5-8| < \varepsilon \\
\Leftrightarrow \quad & |x-3| < \varepsilon
\end{aligned}
$$

$\therefore$ we can choose any δ in the interval $(0, \varepsilon]$ ∎

2) $\lim_{x\to 1} \overbrace{(2x+3)}^{f(x)} = 5$

We need to find $\delta>0$ such that if $0<|x-1|<\delta$, then $|f(x)-5|<\varepsilon$, for all $\varepsilon>0$.

$$
\begin{aligned}
\text{Now,} \quad & |f(x)-5| < \varepsilon \\
\Leftrightarrow \quad & |2x+3-5| < \varepsilon \\
\Leftrightarrow \quad & |2x-2| < \varepsilon \\
\Leftrightarrow \quad & |2(x-1)| < \varepsilon \\
\Leftrightarrow \quad & |2||x-1| < \varepsilon \\
\Leftrightarrow \quad & 2|x-1| < \varepsilon \\
\Leftrightarrow \quad & |x-1| < \frac{\varepsilon}{2}
\end{aligned}
$$

$\therefore$ we can choose any δ in the interval $(0, \frac{\varepsilon}{2}]$ ∎

3) $\lim_{x \to -2} \overbrace{(7-3x)}^{f(x)} = 13$

We need to find $\delta > 0$ such that if $0 < |x-(-2)| < \delta$, then $|f(x) - 13| < \varepsilon$ for all $\varepsilon > 0$.

$0 < |x+2| < \delta$

Now, $|f(x) - 13| < \varepsilon$

$\Leftrightarrow |7-3x-13| < \varepsilon$

$\Leftrightarrow |-3x-6| < \varepsilon$

$\Leftrightarrow |-3(x+2)| < \varepsilon$

$\Leftrightarrow |-3||x+2| < \varepsilon$

$\Leftrightarrow 3|x+2| < \varepsilon$

$\Leftrightarrow |x+2| < \frac{\varepsilon}{3}$

$\therefore$ we can choose any δ in the interval $(0, \frac{\varepsilon}{3})$ ■

4) $\lim_{x \to -1} \overbrace{(5x+2)}^{f(x)} = -3$

We need to find $\delta > 0$ such that if $0 < |x-(-1)| < \delta$, then $|f(x) - (-3)| < \varepsilon$ for all $\varepsilon > 0$.

$0 < |x+1| < \delta$

Now, $|f(x) - (-3)| < \varepsilon$

$\Leftrightarrow |5x+2-(-3)| < \varepsilon$

$\Leftrightarrow |5x+5| < \varepsilon$

$\Leftrightarrow |(5)(x+1)| < \varepsilon$

$\Leftrightarrow |5||x+1| < \varepsilon$

$\Leftrightarrow |x+1| < \frac{\varepsilon}{5}$

$\therefore$ we can choose any δ in the interval $(0, \frac{\varepsilon}{5}]$

■

5) $\lim_{x \to 5} \overbrace{\left(4 - \frac{x}{2}\right)}^{f(x)} = \frac{3}{2}$

We need to find $\delta > 0$ such that if $0 < |x-5| < \delta$, then $|f(x) - \frac{3}{2}| < \varepsilon$ for all $\varepsilon > 0$.

Now, $|f(x) - \frac{3}{2}| < \varepsilon$

$\Leftrightarrow |4 - \frac{x}{2} - \frac{3}{2}| < \varepsilon$

$\Leftrightarrow |-\frac{x}{2} + \frac{5}{2}| < \varepsilon$

$\Leftrightarrow |-\frac{1}{2}(x-5)| < \varepsilon$

$\Leftrightarrow |-\frac{1}{2}||x-5| < \varepsilon$

$\Leftrightarrow \frac{1}{2}|x-5| < \varepsilon$

$\Leftrightarrow |x-5| < 2\varepsilon$

$\therefore$ we can choose any δ in the interval $(0, 2\varepsilon]$ ■

6) $\lim_{x \to 10} \overbrace{15}^{f(x)} = 15$

We need to find $\delta > 0$ such that if $0 < |x-10| < \delta$, then $|f(x) - 15| < \varepsilon$ for all $\varepsilon > 0$.

Now $|f(x) - 15| < \varepsilon$

$\Leftrightarrow |15 - 15| < \varepsilon$

$\Leftrightarrow |0| < \varepsilon$

True for all $\varepsilon > 0$

$\therefore$ we can choose any δ in the interval $(0, \infty)$ ■

7) $\lim_{x\to 0} \overbrace{\left(\frac{2}{3}x+5\right)}^{f(x)} = 5$

We need to find $\delta>0$ such that if $0<|x-0|<\delta$, then $|f(x)-5|<\varepsilon$ for all $\varepsilon>0$.

$$
\begin{aligned}
\text{Now, } \quad & |f(x)-5|<\varepsilon \\
\Leftrightarrow\ & \left|\frac{2}{3}x+5-5\right|<\varepsilon \\
\Leftrightarrow\ & \left|\frac{2}{3}x\right|<\varepsilon \\
\Leftrightarrow\ & \left|\left(\frac{2}{3}\right)(x-0)\right|<\varepsilon \\
\Leftrightarrow\ & \left|\frac{2}{3}\right||x-0|<\varepsilon \\
\Leftrightarrow\ & |x-0|<\frac{3\varepsilon}{2}
\end{aligned}
$$

$\therefore$ we can choose any δ in the interval $\left(0, \frac{3\varepsilon}{2}\right]$ ■

8) $\lim_{x\to 1} \overbrace{\left(-\frac{4}{3}x-2\right)}^{f(x)} = -\frac{10}{3}$

We need to find $\delta>0$ such that if $0<|x-1|<\delta$, then $|f(x)-(-\frac{10}{3})|<\varepsilon$ for all $\varepsilon>0$.

$$
\begin{aligned}
\text{Now } \quad & \left|f(x)-\left(-\frac{10}{3}\right)\right|<\varepsilon \\
\Leftrightarrow\ & \left|-\frac{4}{3}x-2+\frac{10}{3}\right|<\varepsilon \\
\Leftrightarrow\ & \left|-\frac{4}{3}x+\frac{4}{3}\right|<\varepsilon \\
\Leftrightarrow\ & \left|\left(-\frac{4}{3}\right)(x-1)\right|<\varepsilon \\
\Leftrightarrow\ & \left|-\frac{4}{3}\right||x-1|<\varepsilon \\
\Leftrightarrow\ & \frac{4}{3}|x-1|<\varepsilon \\
\Leftrightarrow\ & |x-1|<\frac{3\varepsilon}{4}
\end{aligned}
$$

$\therefore$ we can choose any δ in the interval $\left(0, \frac{3\varepsilon}{4}\right]$ ■

Hilroy