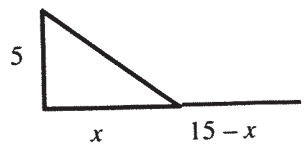
18.



$$C = (5^2 + x^2)^{\frac{1}{2}}(100\,000) + (15 - x)(75\,000)$$

$$= (25 + x^2)^{\frac{1}{2}}(100\,000) + 1\,125\,000 - 75\,000x$$

$$\frac{dC}{dx} = \frac{1}{2}(25 + x^2)^{-\frac{1}{2}}(2x)(100000) - 75\,000$$

Set $\frac{dC}{dx} = 0$

$$100\,000x = 75\,000(25 + x^2)^{\frac{1}{2}}$$

$$\frac{4}{3}x = (25 + x^2)^{\frac{1}{2}}$$

$$\frac{16}{9}x^2 = 25 + x^2$$

$$\frac{16}{9}x^2 - x^2 = 25$$

$$\frac{7}{9}x^2 = 25$$

$$x^2 = \frac{225}{7}$$

$$x = \frac{15}{\sqrt{7}}$$

$$x \doteq 5.67$$

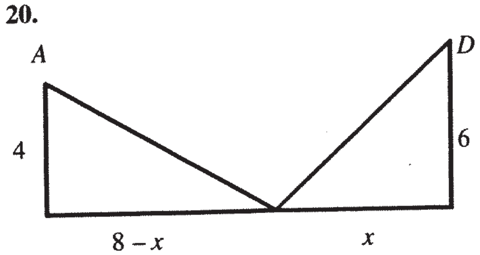
∴ The pipeline should reach land 5.67 km down the shoreline.

19. Consider the distance D, from (x, y) on the circle to $(2, 0)$. Let $d = D^2$

$$d = (x - 2)^2 + y^2$$
$$= (x - 2)^2 + 1 - x^2$$
$$= x^2 - 4x + 4 + 1 - x^2$$
$$= 5 - 4x$$

x is defined only on $[-1, 1]$ and $5x - 4$ is minimized at $x = 1$. So $(1, 0)$ is the closest point.

20.



$$L = (4^2 + (8 - x)^2)^{\frac{1}{2}} + (6^2 + x^2)^{\frac{1}{2}}$$

$$= (16 + 64 - 16x + x^2)^{\frac{1}{2}} + (36 + x^2)^{\frac{1}{2}}$$

$$= (80 - 16x + x^2)^{\frac{1}{2}} + (36 + x^2)^{\frac{1}{2}}$$

$$\frac{dL}{dx} = \frac{1}{2}(80 - 16x + x^2)^{-\frac{1}{2}}(-16 + 2x) + \frac{1}{2}(36 + x^2)^{-\frac{1}{2}}(2x)$$

Set $\frac{dL}{dx} = 0$

$$0 = \frac{x - 8}{(80 - 16x + x^2)^{\frac{1}{2}}} + \frac{x}{(36 + x^2)^{\frac{1}{2}}}$$

$$0 = (x - 8)(36 + x^2)^{\frac{1}{2}} + x(80 - 16x + x^2)^{\frac{1}{2}}$$

$$x(80 - 16x + x^2)^{\frac{1}{2}} = (8 - x)(36 + x^2)^{\frac{1}{2}}$$

$$80x^2 - 16x^3 + x^4 = (64 - 16x + x^2)(36 + x^2)$$

$$80x^2 - 16x^3 + x^4 = 2304 + 64x^2 - 576x - 16x^3 + 36x^2 + x^4$$

$$0 = 20x^2 - 576x + 2304$$

$$0 = 5x^2 - 144x + 576$$

$$x = \frac{144 \pm \sqrt{(-144)^2 - 4(5)(576)}}{2(5)}$$

$$x = \frac{144 \pm \sqrt{9216}}{10}$$

$$x = \frac{72 \pm 48}{5}$$

$x = 4.8$ or $x = 24$ (rejected)

The rest stop should be 4.8 km from Point C and 3.2 km from Point D, or 5.12 km from Ancaster and 7.68 km from Dundas.

6.5 Exercises, page 478

1. $f(x) = (x - 1)^{\frac{1}{3}}$

Domain $= x \in \mathbf{R}$

x-intercept:

$$0 = (x - 1)^{\frac{1}{3}}$$
$$0 = x - 1$$
$$x = 1$$

y-intercept:

$$y = (0 - 1)^{\frac{1}{3}}$$
$$y = -1$$

vertical asymptotes: none; horizontal asymptotes: none

$$\lim_{x \to \infty}(x - 1)^{\frac{1}{3}} = +\infty$$

$$\lim_{x \to -\infty}(x - 1)^{\frac{1}{3}} = -\infty$$

Critical Numbers, Intervals of Increase or Decrease, Local Extrema

$$f(x) = (x - 1)^{\frac{1}{3}}$$

$$f'(x) = \frac{1}{3}(x - 1)^{-\frac{2}{3}}$$

Set $f'(x) = 0$

$$0 = \frac{1}{3}(x - 1)^{-\frac{2}{3}}$$

no solution, $x \neq 1$

	$x<1$	$x>1$
$\frac{1}{3}(x-1)^{-\frac{2}{3}}$	+	+
$f(x)$	increasing	increasing

There are no critical numbers or local extrema for this function, the function is always increasing.

Concavity, Points of Inflection

$f'(x)=\frac{1}{3}(x-1)^{-\frac{2}{3}}$

$f''(x)=\frac{1}{3}\left(-\frac{2}{3}\right)(x-1)^{-\frac{5}{3}}$

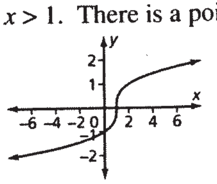
Set $f''(x)=0$

$0=-\frac{2}{3}(x-1)^{-\frac{5}{3}}$

no solution, $x\neq 1$

	$x<1$	$x>1$
$-\frac{2}{9}(x-1)^{-\frac{5}{3}}$	+	−
$f(x)$	concave up	concave down
	point of inflection at $x=1$	

The function is concave up when $x<1$ and concave down when
$x>1$. There is a point of inflection at $(x, y)=(1, 0)$



2. $f(x)=(x-4)^{\frac{2}{3}}$

Domain $= x\in \mathbf{R}$

x-intercept

$0=(x-4)^{\frac{2}{3}}$

$0=x-4$

$x=4$

y-intercept

$y=(0-4)^{\frac{2}{3}}$

$y=2\sqrt[3]{2}$

vertical asymptotes: none; horizontal asymptotes: none

$\lim_{x\to\infty}(x-4)^{\frac{2}{3}}=+\infty$

$\lim_{x\to-\infty}(x-4)^{\frac{2}{3}}=+\infty$

Critical Numbers, Intervals of Increase or Decrease, Local Extrema

$f(x)=(x-4)^{\frac{2}{3}}$

$f'(x)=\frac{2}{3}(x-4)^{-\frac{1}{3}}$

Set $f'(x)=0$

$0=\frac{2}{3}(x-4)^{-\frac{1}{3}}$

no solution, $x\neq 4$

	$x<4$	$x>4$
$\frac{2}{3}(x-4)^{-\frac{1}{3}}$	−	+
$f(x)$	decreasing	increasing
	local minimum at $x=4$	

There are no critical numbers for this function. There is a local minimum at $(x, y)=(4, 0)$. The function is decreasing if $x<4$ and increasing if $x>4$.

Concavity, Points of Inflection

$f'(x)=\frac{2}{3}(x-4)^{-\frac{1}{3}}$

$f''(x)=\frac{2}{3}\left(-\frac{1}{3}\right)(x-4)^{-\frac{4}{3}}$

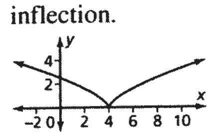
Set $f''(x)=0$

$0=-\frac{2}{9}(x-4)^{-\frac{4}{3}}$

no solution, $x\neq 4$

	$x<4$	$x>4$
$-\frac{2}{9}(x-4)^{-\frac{5}{3}}$	−	−
$f(x)$	concave down	concave down

The function is always concave down. There are no points of inflection.



3. $f(x)=\sqrt{x+5}$

Domain $=\{x \mid x\geq -5, x\in \mathbf{R}\}$

x-intercept

$0=\sqrt{x+5}$

$0=x+5$

$x=-5$

y-intercept

$y=\sqrt{0+5}$

$y=\sqrt{5}$

vertical asymptotes: none; horizontal asymptotes: none

$\lim_{x\to\infty}\sqrt{x+5}=+\infty$

Critical Numbers, Intervals of Increase or Decrease, Local Extrema

$f(x) = \sqrt{x+5}$

$f'(x) = \frac{1}{2}(x+5)^{-\frac{1}{2}}$

Set $f'(x) = 0$

$0 = \frac{1}{2}(x+5)^{-\frac{1}{2}}$

no solution, $x \neq -5$

	$x < -5$	$x > -5$
$\frac{1}{2}(x+5)^{-\frac{1}{2}}$	undefined	+
$f(x)$	undefined	increasing

There are no critical numbers or local turning points for this function. The function is increasing if $x > -5$.

Concavity, Points of Inflection

$f'(x) = \frac{1}{2}(x+5)^{-\frac{1}{2}}$

$f''(x) = \frac{1}{2}\left(-\frac{1}{2}\right)(x+5)^{-\frac{3}{2}}$

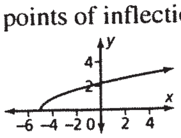
Set $f''(x) = 0$

$0 = -\frac{1}{4}(x+5)^{-\frac{3}{2}}$

no solution, $x \neq -5$

	$x < -5$	$x > -5$
$-\frac{1}{4}(x+5)^{-\frac{3}{2}}$	undefined	−
$f(x)$	undefined	concave down

The function is concave down when $x > -5$. There are no points of inflection.



4. $f(x) = \sqrt{(x+3)^2}$

Domain $= x \in \mathbf{R}$

x-intercept

$0 = \sqrt{(x+3)^2}$

$0 = x+3$

$x = -3$

y-intercept

$y = \sqrt{(0+3)^2}$

$y = 3$

vertical asymptotes: none; horizontal asymptotes: none

$\lim_{x\to\infty} \sqrt{(x+3)^2} = +\infty$

$\lim_{x\to-\infty} \sqrt{(x+3)^2} = +\infty$

Critical Numbers, Intervals of Increase or Decrease, Local Extrema

$f(x) = \sqrt{(x+3)^2}$

$f'(x) = \frac{1}{2}\left((x+3)^2\right)^{-\frac{1}{2}}(2)(x+3)$

$= (x+3)^{-1}(x+3)$

$= 1, \quad x \neq -3$

Set $f'(x) = 0$

$0 = 1$

no solution, $x \neq -3$

	$x < -3$	$x > -3$
1	+	+
$f(x)$	increasing	increasing

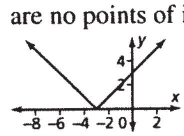
There are no critical numbers or local turning points for this function. The function is increasing when $x \neq -3$.

Concavity, Points of Inflection

$f'(x) = 1$

$f''(x) = 0$

The function is neither concave up or concave down. There are no points of inflection.



5. $f(x) = \frac{15}{x+3}$

Domain $= \{x \mid x \neq -3, x \in \mathbf{R}\}$

x-intercept

$0 = \frac{15}{x+3}$

no solution, no x-intercept

y-intercept

$y = \frac{15}{0+3}$

$y = 5$

vertical asymptotes

$x + 3 = 0$

$x = -3$

horizontal asymptotes

$\lim_{x\to\infty} \frac{15}{x+3} = 0$

$\lim_{x\to-\infty} \frac{15}{x+3} = 0$

$y = 0$

Critical Numbers, Intervals of Increase or Decrease, Local Extrema

$f(x) = \frac{15}{x+3}$

$f'(x) = -\frac{15}{(x+3)^2}$

Set $f'(x) = 0$

$0 = -\frac{15}{(x+3)^2}$

no solution, $x \neq -3$

	$x < -3$	$x > -3$
$-\frac{15}{(x+3)^2}$	–	–
$f(x)$	decreasing	decreasing

There are no critical numbers or local turning points for this function. The function is decreasing when $x \neq -3$.

Concavity, Points of Inflection

$$f'(x) = -\frac{15}{(x+3)^2}$$

$$f''(x) = \frac{15(2)}{(x+3)^3}$$

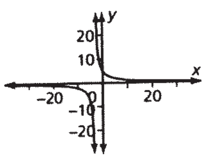
$$= \frac{30}{(x+3)^3}$$

Set $f''(x) = 0$

no solution, $x \neq -3$

	$x < -3$	$x > -3$
$\frac{30}{(x+3)^3}$	–	+
$f(x)$	concave down	concave up

The function is concave down when $x < -3$ and concave up when $x > -3$. There are no points of inflection.



6. $f(x) = (2x - 4)^{-2}$

Domain = $\{x \mid x \neq 2, x \in \mathbf{R}\}$

x-intercept

$0 = (2x - 4)^{-2}$

no solution, no x-intercept

y-intercept

$y = (2(0) - 4)^{-2}$

$y = \frac{1}{16}$

vertical asymptotes

$2x - 4 = 0$

$x = 2$

horizontal asymptotes

$\lim_{x\to\infty} (2x - 4)^{-2} = 0$

$\lim_{x\to-\infty} (2x - 4)^{-2} = 0$

$y = 0$

Critical Numbers, Intervals of Increase or Decrease, Local Extrema

$f(x) = (2x - 4)^{-2}$

$f'(x) = -2(2x - 4)^{-3}(2)$

Set $f'(x) = 0$

$0 = -4(2x - 4)^{-3}$

no solution, $x \neq 2$

	$x < 2$	$x > 2$
$-4(2x - 4)^{-3}$	+	–
$f(x)$	increasing	decreasing

There are no critical numbers or local turning points for this function. The function is increasing when $x < 2$ and decreasing when $x > 2$.

Concavity, Points of Inflection

$f'(x) = -4(2x - 4)^{-3}$

$f''(x) = -4(-3)(2x - 4)^{-4}(2)$

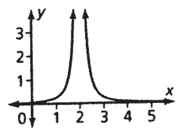
$= 24(2x - 4)^{-4}$

Set $f''(x) = 0$

no solution, $x \neq 2$

	$x < 2$	$x > 2$
$24(2x - 4)^{-4}$	+	+
$f(x)$	concave up	concave up

The function is concave up when $x \neq 2$. There are no points of inflection.



7. $f(x) = (x^3 + x)^2$

Domain = $x \in \mathbf{R}$

x-intercept

$0 = (x^3 + x)^2$

$0 = x^3 + x$

$0 = x(x^2 + 1)$

$x = 0$

y-intercept

$y = (0^3 + 0)^2$

$y = 0$

vertical asymptotes: none; horizontal asymptotes: none

$\lim_{x\to\infty} (x^3 + x)^2 = +\infty$

$\lim_{x\to-\infty} (x^3 + x)^2 = +\infty$

Critical Numbers, Intervals of Increase or Decrease, Local Extrema

$f(x) = (x^3 + x)^2$

$f'(x) = 2(x^3 + x)(3x^2 + 1)$

Set $f'(x) = 0$

$0 = 2(x^3 + x)(3x^2 + 1)$

$0 = 2x(x^2 + 1)(3x^2 + 1)$

$x = 0$

	$x < 0$	$x > 0$
$2x$	–	+
$x^2 + 1$	+	+
$3x^2 + 1$	+	+
$f'(x)$	$(-)(+)(+) = -$	$(+)(+)(+) = +$
$f(x)$	decreasing	increasing
	local minimum at $x = 0$	

This function has a critical point at (0, 0). This point is also a local minimum. This function is decreasing when $x < 0$ while it is increasing when $x > 0$.

Concavity, Points of Inflection

$f'(x) = 2(x^3 + x)(3x^2 + 1)$

$f''(x) = 2[(3x^2 + 1)(3x^2 + 1) + (6x)(x^3 + x)]$

$= 2[(3x^2 + 1)^2 + 6x(x^3 + x)]$

Set $f''(x) = 0$

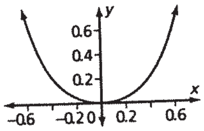
$0 = 2[(3x^2 + 1)^2 + 6x(x^3 + x)]$

$0 = 2(9x^4 + 6x^2 + 1 + 6x^4 + 6x^2)$

$0 = 2(15x^4 + 12x^2 + 1)$

no real roots

The function is always concave up. There are no points of inflection.



8. $f(x) = (x^2 - 9)^2$

Domain = $x \in \mathbf{R}$

x-intercept

$0 = (x^2 - 9)^2$

$0 = x^2 - 9$

$0 = (x - 3)(x + 3)$

$x = -3$ or $x = 3$

y-intercept

$y = (0^2 - 9)^2$

$y = 81$

vertical asymptotes: none, horizontal asymptotes: none

$\lim_{x\to\infty} (x^2 - 9)^2 = +\infty$, $\lim_{x\to-\infty} (x^2 - 9)^2 = +\infty$

Critical Numbers, Intervals of Increase or Decrease, Local Extrema

$f(x) = (x^2 - 9)^2$

$f'(x) = 2(x^2 - 9)(2x)$

Set $f'(x) = 0$

$0 = 4x(x^2 - 9)$

$0 = 4x(x - 3)(x + 3)$

$x = -3$ or $x = 0$ or $x = 3$

	$x < -3$	$-3 < x < 0$	$0 < x < 3$	$x > 3$
$4x$	−	−	+	+
$x - 3$	−	−	−	+
$x + 3$	−	+	+	+
$f'(x)$	(−)(−)(−) = −	(−)(−)(+) = +	(+)(−)(+) = −	(+)(+)(+) = +
$f(x)$	decreasing	increasing	decreasing	increasing
	minimum at $x = -3$	maximum at $x = 0$		minimum at $x = 3$

This function has a critical numbers −3, 0, 3. There are local minimums at (−3, 0) and (3, 0), and a local maximum at (0, 81). This function is decreasing when $x < -3$ and $0 < x < 3$ while it is increasing when $-3 < x < 0$ and $x > 3$.

Concavity, Points of Inflection

$f'(x) = 4x(x^2 - 9)$

$f''(x) = 4(x^2 - 9) + 2x(4x)$

$= 4(3x^2 - 9)$

Set $f''(x) = 0$

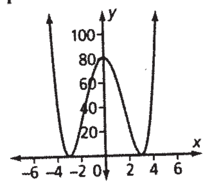
$0 = 12(x^2 - 3)$

$0 = (x - \sqrt{3})(x + \sqrt{3})$

$x = -\sqrt{3}$ or $x = \sqrt{3}$

	$x < -\sqrt{3}$	$-\sqrt{3} < x < \sqrt{3}$	$x > \sqrt{3}$
$x - \sqrt{3}$	−	−	+
$x + \sqrt{3}$	−	+	+
$f'(x)$	(−)(−) = +	(−)(+) = −	(+)(+) = +
$f(x)$	concave up	concave down	concave up
	point of inflection at $x =$ ~~-1~~ $-\sqrt{3}$		point of inflection at $x =$ ~~1~~ $\sqrt{3}$

The function is concave up when $x < -\sqrt{3}$ and when $x > \sqrt{3}$ while it is concave down when $-\sqrt{3} < x < \sqrt{3}$. There are points of inflection at the points $(-\sqrt{3}, 36)$ and $(\sqrt{3}, 36)$.



9. $f(x) = x(x^2 - 12)$

Domain = $x \in \mathbf{R}$

x-intercept

$0 = x(x^2 - 12)$

$x = -\sqrt{12}$ or $x = 0$ or $x = \sqrt{12}$

y-intercept

$y = 0(0^2 - 12)$

$y = 0$

vertical asymptotes: none; horizontal asymptotes: none

$\lim_{x\to\infty} x(x^2 - 12) = +\infty$

$\lim_{x\to-\infty} x(x^2 - 12) = -\infty$

Critical Numbers, Intervals of Increase or Decrease, Local Extrema

$f(x) = x(x^2 - 12)$

$f'(x) = (x^2 - 12) + x(2x)$

Set $f'(x) = 0$

$0 = 3x^2 - 12$

$0 = 3(x^2 - 4)$

$0 = (x - 2)(x + 2)$

$x = -2$ or $x = 2$

	$x < -2$	$-2 < x < 2$	$x > 2$
$x - 2$	−	−	+
$x + 2$	−	+	+
$f'(x)$	(−)(−) = +	(−)(+) = −	(+)(+) = +
$f(x)$	increasing	decreasing	increasing
	maximum at $x = -2$	minimum at $x = 2$	

This function has critical numbers at −2, 2. There is a local minimum at (2, −16) and a local maximum at (−2, 16). This function is decreasing when $-2 < x < 2$ while it is increasing when $x < -2$ and $x > 2$.

Concavity, Points of Inflection

$f'(x) = 3x^2 - 12$

$f''(x) = 6x$

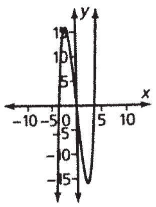
Set $f''(x) = 0$

$0 = 6x$

$x = 0$

	$x < 0$	$x > 0$
x	−	+
$f'(x)$	−	+
$f(x)$	concave down	concave up
	point of inflection at $x = 0$	

The function is concave up when $x > 0$ and concave down when $x < 0$. There is a point of inflection at the point (0, 0).



10. $f(x) = x\sqrt{4-x}$

Domain $= \{x \mid x \le 4, x \in \mathbf{R}\}$

x-intercept

$0 = x\sqrt{4-x}$

$0 = x^2(4-x)$

$x = 0$ or $x = 4$

y-intercept

$y = 0\sqrt{4-0}$

$y = 0$

vertical asymptotes: none; horizontal asymptotes: none

$\lim_{x\to-\infty} x\sqrt{4-x} = -\infty$

Critical Numbers, Intervals of Increase or Decrease, Local Extrema

$f(x) = x\sqrt{4-x}$

$f'(x) = \sqrt{4-x} + \frac{1}{2}(4-x)^{-\frac{1}{2}}(-1)(x)$

Set $f'(x) = 0$

$0 = \sqrt{4-x} + \frac{1}{2}(4-x)^{-\frac{1}{2}}(-1)(x)$

$0 = \frac{8-2x-x}{2\sqrt{4-x}}$

$0 = \frac{8-3x}{2\sqrt{4-x}}$

$3x = 8$

$x = \frac{8}{3}$

	$x < \frac{8}{3}$	$\frac{8}{3} < x < 4$
$8 - 3x$	+	−
$2\sqrt{4-x}$	+	+
$f'(x)$	$\frac{+}{+} = +$	$\frac{-}{+} = -$
$f(x)$	increasing	decreasing
	maximum at $x = \frac{8}{3}$	

This function has a critical number at $x = \frac{8}{3}$. There is a local maximum at $\left(\frac{8}{3}, \frac{16\sqrt{3}}{9}\right)$. This function is decreasing when $\frac{8}{3} < x < 4$ while it is increasing when $x < \frac{8}{3}$.

Concavity, Points of Inflection

$f'(x) = \frac{8-3x}{2\sqrt{4-x}}$

$f''(x) = \frac{1}{2}\left[\frac{-3\left(\sqrt{4-x}\right) - \frac{1}{2}\left(\frac{1}{\sqrt{4-x}}\right)(-1)(8-3x)}{(4-x)}\right]$

$= \frac{-3\sqrt{4-x}}{2(4-x)} + \frac{8-3x}{4(4-x)\left(\sqrt{4-x}\right)}$

Set $f''(x) = 0$

$0 = \frac{-3\sqrt{4-x}}{2(4-x)} + \frac{8-3x}{4(4-x)\left(\sqrt{4-x}\right)}$

$0 = \frac{-6(4-x)+8-3x}{4(4-x)\left(\sqrt{4-x}\right)}$

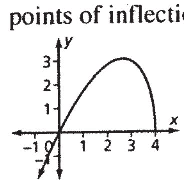
$0 = \frac{-16+3x}{4(4-x)\left(\sqrt{4-x}\right)}$

$3x = 16$

$x = \frac{16}{3}$

but $x \le 4$

The function is concave down when $x \le 4$. There are no points of inflection.



11. $f(x) = \frac{x}{(x-2)^2}$

Domain $= \{x \mid x \ne 2, x \in \mathbf{R}\}$

x-intercept

$0 = \frac{x}{(x-2)^2}$

$x = 0$

y-intercept

$y = \frac{0}{(0-2)^2}$

$y = 0$

vertical asymptotes

$0 = (x-2)^2$

$0 = x - 2$

$x = 2$

horizontal asymptotes

$\lim_{x\to\infty} \frac{x}{(x-2)^2} = 0$

$\lim_{x\to-\infty} \frac{x}{(x-2)^2} = 0$

$y = 0$

Critical Numbers, Intervals of Increase or Decrease, Local Extrema

$f(x) = \frac{x}{(x-2)^2}$

$f'(x) = \frac{(x-2)^2 - x(2)(x-2)}{(x-2)^4}$

$= \frac{-x^2+4}{(x-2)^4}$

Set $f'(x) = 0$

$0 = -x^2 + 4$

$0 = (-x+2)(x+2)$

$x = -2$ or $x = 2$ but $x \neq 2$

	$x < -2$	$-2 < x < 2$	$x > 2$
$-x^2+4$	$-$	$+$	$-$
$(x-2)^4$	$+$	$+$	$+$
$f'(x)$	$\frac{-}{+} = -$	$\frac{+}{+} = +$	$\frac{-}{+} = -$
$f(x)$	decreasing	increasing	decreasing
	minimum at $x = -2$	undefined at $x = 2$	

The critical numbers for this function are -2 and 2. There is a local minimum at the point $\left(-2, -\frac{1}{8}\right)$. The function is increasing if $-2 < x < 2$ while it is decreasing when $x < -2$ and $x > 2$.

Concavity, Points of Inflection

$$f'(x) = \frac{-x^2+4}{(x-2)^4}$$

$$f''(x) = \frac{-2x(x-2)^4 - 4(x-2)^3(-x^2+4)}{(x-2)^8}$$

$$= \frac{-2(x-2)^3\left(x(x-2) + 2(-x^2+4)\right)}{(x-2)^8}$$

$$= \frac{-2(-x^2-2x+8)}{(x-2)^5}, \quad x \neq 2$$

Set $f''(x) = 0$

$0 = -x^2 - 2x + 8$

$0 = -x^2 - 4x + 2x + 8$

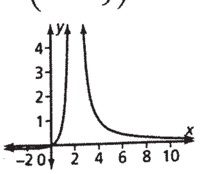
$0 = -x(x+4) + 2(x+4)$

$0 = (x+4)(2-x)$

$x = -4$ or $x = 2$, but $x \neq 2$

	$x < -4$	$-4 < x < 2$	$x > 2$
$x+4$	$-$	$+$	$+$
$2-x$	$+$	$+$	$-$
$\frac{-2(x-2)^3}{(x-2)^8}$	$+$	$+$	$-$
$f''(x)$	$(-)(+)(+) = -$	$(+)(+)(+) = +$	$(+)(-)(-) = +$
$f(x)$	concave down	concave up	concave up
	point of inflection at $x = -4$	undefined at $x = 2$	

The function is concave up when $-4 < x < 2$ and $x > 2$ while it is concave down when $x < -4$. There is a point of inflection at $\left(-4, -\frac{1}{9}\right)$.



12. $f(x) = \frac{x}{\sqrt{x^2-1}}$

Domain $= \{x \mid x < -1 \text{ or } x > 1, x \in \mathbf{R}\}$

x-intercepts

$0 = \frac{x}{\sqrt{x^2-1}}$

$x = 0$

but $x < -1$ or $x > 1$, no x-int

y-intercept

$y = \frac{0}{\sqrt{0^2-1}}$

negative root, no y-intercept

vertical asymptotes

$0 = \sqrt{x^2-1}$

$0 = x^2 - 1$

$x = -1$ or $x = 1$

note: x is undefined for $-1 \le x \le 1$

horizontal asymptotes

$$\lim_{x\to\infty} \frac{x}{\sqrt{x^2-1}} = 1$$

$$\lim_{x\to-\infty} \frac{x}{\sqrt{x^2-1}} = -1$$

$y = -1$, $y = 1$

Critical Numbers, Intervals of Increase or Decrease, Local Extrema

$$f(x) = \frac{x}{\sqrt{x^2-1}}$$

$$f(x) = \frac{x}{(x^2-1)^{\frac{1}{2}}}$$

$$f'(x) = \frac{(x^2-1)^{\frac{1}{2}} - x\left(\frac{1}{2}\right)(x^2-1)^{-\frac{1}{2}}(2x)}{x^2-1}$$

$$= \frac{x^2-1-x^2}{(x^2-1)^{\frac{3}{2}}}$$

$$= -\frac{1}{(x^2-1)^{\frac{3}{2}}}$$

Set $f'(x) = 0$

$$0 = -\frac{1}{(x^2-1)^{\frac{3}{2}}}$$

no solution but $x \neq -1, 1$

	$x < -1$	$-1 < x < 1$	$x > 1$
$-\frac{1}{(x^2-1)^{\frac{3}{2}}}$	$-$	undefined	$-$
$f'(x)$	$-$	undefined	$-$
$f(x)$	decreasing	undefined	decreasing

There are no critical numbers or local maximums or minimums for this function. The function is decreasing if $x <$ -1 and $x > 1$.

Concavity, Points of Inflection

$$f'(x) = -\frac{1}{\left(x^2-1\right)^{\frac{3}{2}}}$$

$$f''(x) = \frac{3(2x)}{2\left(x^2-1\right)^{\frac{5}{2}}}$$

$$= \frac{3x}{\left(x^2-1\right)^{\frac{5}{2}}}$$

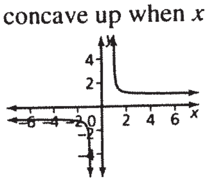
Set $f''(x) = 0$

$0 = 3x$

$x = 0$, but $x \neq 0$; x is undefined for $-1 \leq x \leq 1$

	$x < -1$	$-1 < x < 0$	$0 < x < 1$	$x > 1$
$\frac{3x}{\left(x^2-1\right)^{\frac{5}{2}}}$	$-$	undefined	undefined	$+$
$f''(x)$	$-$	undefined	undefined	$+$
$f(x)$	concave down	undefined	undefined	concave up

The function is concave down when $x < -1$ while it is concave up when $x > 1$. There are no points of inflection.



13. $f(x) = \frac{(x-1)^2}{(x+1)^3}$

Domain $= \{x \mid x \neq -1, x \in \mathbf{R}\}$

x-intercept

$0 = \frac{(x-1)^2}{(x+1)^3}$

$0 = x - 1$

$x = 1$

y-intercept

$y = \frac{(0-1)^2}{(0+1)^3}$

$y = 1$

vertical asymptotes

$0 = (x+1)^3$

$0 = x + 1$

$x = -1$

horizontal asymptotes

$$\lim_{x\to\infty} \frac{(x-1)^2}{(x+1)^3} = 0$$

$$\lim_{x\to-\infty} \frac{(x-1)^2}{(x+1)^3} = 0$$

$y = 0$

Critical Numbers, Intervals of Increase or Decrease, Local Extrema

$$f(x) = \frac{(x-1)^2}{(x+1)^3}$$

$$f'(x) = \frac{2(x-1)(x+1)^3 - 3(x+1)^2(x-1)^2}{(x+1)^6}$$

$$= \frac{(x-1)(x+1)^2(2x+2-3x+3)}{(x+1)^6}$$

$$= \frac{(x-1)(5-x)}{(x+1)^4}, \quad x \neq 1$$

Set $f'(x) = 0$

$0 = (x-1)(5-x)$

$x = 1$ or $x = 5$

	$x < -1$	$-1 < x < 1$	$1 < x < 5$	$x > 5$
$x - 1$	$-$	$-$	$+$	$+$
$5 - x$	$+$	$+$	$+$	$-$
$(x+1)^4$	$+$	$+$	$+$	$+$
$f'(x)$	$\frac{(-)(+)}{(+)} = -$	$\frac{(-)(+)}{+} = -$	$\frac{(+)(+)}{+} = +$	$\frac{(+)(-)}{+} = -$
$f(x)$	decreasing	decreasing	increasing	decreasing
	undefined at $x = -1$	minimum at $x = 1$	maximum at $x = 5$	

The critical numbers for this function are 1 and 5. There is a local minimum at the point $(1, 0)$ and a local maximum at the point $\left(5, \frac{2}{27}\right)$. The function is increasing if $1 < x < 5$ while it is decreasing when $x < -1$, $-1 < x < 1$ and $x > 5$.

Concavity, Points of Inflection

$$f'(x) = \frac{-x^2+6x-5}{(x+1)^4}$$

$$f''(x) = \frac{(-2x+6)(x+1)^4 - 4(x+1)^3\left(-x^2+6x-5\right)}{(x+1)^8}$$

$$= \frac{(x+1)^3\left((-2x+6)(x+1) - 4\left(-x^2+6x-5\right)\right)}{(x+1)^8}$$

$$= \frac{-2x^2+4x+6+4x^2-24x+20}{(x+1)^5}, \quad x \neq -1$$

$$= \frac{2\left(x^2-10x+13\right)}{(x+1)^5}$$

Set $f''(x) = 0$

$0 = x^2 - 10x + 13$

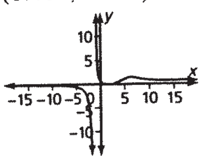
$$x = \frac{10 \pm \sqrt{(-10)^2 - 4(1)(13)}}{2(1)}$$

$$x = \frac{10 \pm \sqrt{48}}{2}$$

$x \doteq 1.536$ or $x \doteq 8.464$

	$x < -1$	$-1 < x < 1.536$	$1.536 < x < 8.464$	$x > 8.464$
$\frac{2(x^2-10x+13)}{(x+1)^5}$	−	+	−	+
$f''(x)$	−	+	−	+
$f(x)$	concave down	concave up	concave down	concave up
	undefined at $x = -1$	point of inflection at $x = 1.536$		point of inflection at $x = 8.464$

The function is concave up when $-1 < x < 1.536$ and $x > 8.464$ while it is concave down when $x < -1$ and $1.536 < x < 8.464$. There are two points of inflection (1.536, 0.018) and (8.464, 0.066).



14. $f(x) = (x^2 + 1)^2(x^2 - 1)^3$

Domain = $x \in \mathbf{R}$

x-intercept

$0 = (x^2 + 1)^2(x^2 - 1)^3$

$0 = (x^2 + 1)^2(x - 1)^3(x + 1)^3$

$x = -1$ or $x = 1$

y-intercept

$y = (0^2 + 1)^2(0^2 - 1)^3$

$y = -1$

vertical asymptotes: none; horizontal asymptotes: none

$\lim_{x\to\infty} (x^2 + 1)^2(x^2 - 1)^3 = +\infty$

$\lim_{x\to-\infty} (x^2 + 1)^2(x^2 - 1)^3 = +\infty$

Critical Numbers, Intervals of Increase or Decrease, Local Extrema

$f(x) = (x^2 + 1)^2(x^2 - 1)^3$

$f'(x) = 2(x^2 + 1)(2x)(x^2 - 1)^3 + 3(x^2 - 1)^2(2x)(x^2 + 1)^2$

$= (2x)(x^2 + 1)(x^2 - 1)^2(2x^2 - 2 + 3x^2 + 3)$

$= 2x(x^2 + 1)(x^2 - 1)^2(5x^2 + 1)$

Set $f'(x) = 0$

$0 = 2x(x^2 + 1)(x^2 - 1)^2(5x^2 + 1)$

$x = -1$ or $x = 0$ or $x = 1$

	$x < -1$	$-1 < x < 0$	$0 < x < 1$	$x > 1$
$2x$	−	−	+	+
$(x^2 + 1)$	+	+	+	+
$(x^2 - 1)^2$	+	+	+	+
$(5x^2 + 1)$	+	+	+	+
$f'(x)$	(−)(+)(+)(+) = −	(−)(+)(+)(+) = −	(+)(+)(+)(+) = +	(+)(+)(+)(+) = +
$f(x)$	decreasing	decreasing	increasing	increasing
	neither a max nor min at $x = -1$	minimum at $x = 0$		neither a max nor min at $x = 1$

The critical numbers for this function are −1, 0, and 1. There is a local minimum at the point (0, −1). The function is increasing when $x > 0$ while it is decreasing when $x < 0$.

Concavity, Points of Inflection

$f'(x) = 2x(x^2 + 1)(x^2 - 1)^2(5x^2 + 1)$

$= 2x(5x^4 + 6x^2 + 1)(x^2 - 1)^2$

$= (10x^5 + 12x^3 + 2x)(x^2 - 1)^2$

$f''(x) = (50x^4 + 36x^2 + 2)(x^2 - 1)^2$

$+ 2(x^2 - 1)(2x)(10x^5 + 12x^3 + 2x)$

$= 2(x^2 - 1)^2(25x^4 + 18x^2 + 1) + 8x^2(x^2 - 1)(5x^4 + 6x^2 + 1)$

$= 2(x^2 - 1)[(x^2 - 1)(25x^4 + 18x^2 + 1) + 4x^2(5x^4 + 6x^2 + 1)]$

Set $f''(x) = 0$

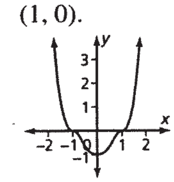
$0 = 2(x^2 - 1)(45x^6 + 17x^4 - 13x^2 - 1)$

$0 = 2(x - 1)(x + 1)(45x^6 + 17x^4 - 13x^2 - 1)$

$x = -1$ or $x \doteq -0.6519$ or $x \doteq 0.6519$ or $x = 1$

	$x < -1$	$-1 < x < -0.6519$	$-0.6519 < x < 0.6519$	$0.6519 < x < 1$	$x > 1$
$x - 1$	−	−	−	+	+
$x + 1$	−	+	+	+	+
$45x^6 + 17x^4 - 13x^2 - 1$	+	−	−	−	+
$f''(x)$	(−)(−)(+) = +	(−)(+)(−) = −	(−)(+)(−) = +	(+)(+)(−) = −	(+)(+)(+) = +
$f(x)$	concave up	concave down	concave up	concave down	concave up
	point of inflection at $x = -1$	point of inflection at $x = -0.6519$	point of inflection at $x = 0.6519$	point of inflection at $x = 1$	

The function is concave up when $x < -1$, $-0.6519 < x < 0.6519$ and $x > 1$ while it is concave down when $-1 < x < -0.6519$ and $0.6519 < x < 1$. There are four points of inflection: (−1, 0), (−0.6519, −0.3861), (0.6519, −0.3861), and (1, 0).



15. $f(x) = \left(\frac{x-2}{x+3}\right)^2$

Domain = $\{x \mid x \neq -3, x \in \mathbf{R}\}$

x-intercept

$0 = \left(\frac{x-2}{x+3}\right)^2$

$x = 2$

y-intercept

$y = \left(\frac{0-2}{0+3}\right)^2$

$y = \frac{4}{9}$

vertical asymptotes

$0 = (x + 3)^2$

$0 = x + 3$

$x = -3$

horizontal asymptotes

$\lim_{x\to\infty} \left(\frac{x-2}{x+3}\right)^2 = 1$

$\lim_{x \to -\infty} \left(\frac{x-2}{x+3}\right)^2 = 1$

$y = 1$

Critical Numbers, Intervals of Increase or Decrease, Local Extrema

$f(x) = \left(\frac{x-2}{x+3}\right)^2 = \frac{(x-2)^2}{(x+3)^2}$

$$f'(x) = \frac{2(x-2)(x+3)^2 - 2(x+3)(x-2)^2}{(x+3)^4}$$
$$= \frac{2(x-2)(x+3)(x+3-x+2)}{(x+3)^4}$$
$$= \frac{10(x-2)}{(x+3)^3}, \quad x \neq -3$$

Set $f'(x) = 0$

$0 = x - 2$

$x = 2$

	$x < -3$	$-3 < x < 2$	$x > 2$
$x - 2$	$-$	$-$	$+$
$(x+3)^3$	$-$	$+$	$+$
$f'(x)$	$\frac{-}{-} = +$	$\frac{-}{+} = -$	$\frac{+}{+} = +$
$f(x)$	increasing	decreasing	increasing
	undefined at $x = -3$	minimum at $x = 2$	

The critical number for this function is 2. There is a local minimum at the point (2, 0). The function is increasing if $x < -3$ and $x > 2$ while it is decreasing when $-3 < x < 2$.

Concavity, Points of Inflection

$f'(x) = \frac{10(x-2)}{(x+3)^3}$

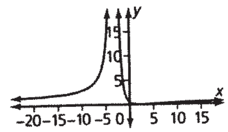
$$f''(x) = 10\left[\frac{(x+3)^3 - 3(x+3)^2(x-2)}{(x+3)^6}\right]$$
$$= 10\left[\frac{(x-3)^2(x+3-3x+6)}{(x+3)^6}\right]$$
$$= 10\left[\frac{-2x+9}{(x+3)^4}\right], \quad x \neq -3$$

Set $f''(x) = 0$

$x = \frac{9}{2}$

	$x < -3$	$-3 < x < \frac{9}{2}$	$x > \frac{9}{2}$
$\frac{-2x+9}{(x+3)^4}$	$+$	$+$	$-$
$f''(x)$	$+$	$+$	$-$
$f(x)$	concave up	concave up	concave down
	undefined at $x = -3$	point of inflection at $x = \frac{9}{2}$	

The function is concave up when $x < -3$ and $-3 < x < \frac{9}{2}$ while it is concave down when $x > \frac{9}{2}$. There is a point of inflection at $\left(\frac{9}{2}, \frac{1}{9}\right)$.



6.6 Exercises, page 483

1. **(a)** $\frac{d}{dx}(x^3) = 3x^2$ **(b)** $\frac{d}{dx}(5x^4) = 20x^3$

(c) $\frac{d}{dx}(y) = \frac{dy}{dx}$ **(d)** $\frac{d}{dx}(y^3) = 3y^2\frac{dy}{dx}$

(e) $\frac{d}{dx}(5y^6) = 30y^5\frac{dy}{dx}$

2. **(a)** $\frac{d}{dx}(xy) = y + x\frac{dy}{dx}$ **(b)** $\frac{d}{dx}(2x^4y) = 8x^3y + 2x^4\frac{dy}{dx}$

(c) $\frac{d}{dx}(x^2y^2) = 2xy^2 + 2x^2y\frac{dy}{dx}$

(d) $\frac{d}{dx}(5x^2y^3) = 10xy^3 + 15x^2y^2\frac{dy}{dx}$

(e) $\frac{d}{dx}(-2x^4y^6) = -8x^3y^6 - 12x^4y^5\frac{dy}{dx}$

3. $2p + 7c = 8$

$2\frac{dp}{dc} + 7 = 0$

$\frac{dp}{dc} = -\frac{7}{2}$

4. $xy = 1$

explicitly

$y = \frac{1}{x}$

$\frac{dy}{dx} = -\frac{1}{x^2}$

(by power or quotient rule)

implicitly

$xy = 1$

$y + x\frac{dy}{dx} = 0$

$\frac{dy}{dx} = -\frac{y}{x}$

$= -\frac{\frac{1}{x}}{x}$

$= -\frac{1}{x^2}$

5. **(a)** $x^2 + y^2 = 4$

$2x + 2y\frac{dy}{dx} = 0$

$\frac{dy}{dx} = -\frac{x}{y}$