

SOLVING QUADRATIC TRIGONOMETRIC EQUATIONS
SUPPLEMENTARY PROBLEMS

1) Solve each of the following for $0^\circ \leq x \leq 360^\circ$.

a) $3\sin^2 x - 8\sin x - 3 = 0$ b) $5\cos^2 x + 6\cos x = 8$ c) $\cot x \cos x = \cos x$

d) $\sin^2 x + 2\sin x - 2 = 0$ e) $\cos x \csc x = 2\cos x$

f) $4\sin x \tan x - 3\tan x + 20\sin x - 15 = 0$ (Hint: one factor is $\tan x + 5$)

2) Solve each of the following (using radian measure) for $0 \leq x \leq 2\pi$.

a) $\sin^2 x - 2\sin x - 3 = 0$ b) $\sin^2 x = 6\sin x - 9$ c) $6\cos^2 x + \cos x - 1 = 0$

d) $\sec^2 x - 2\tan^2 x = 0$ e) $\sec^2 x + \tan x = 3$ f) $\sin 2x = \cos x$

3) A model for a bouncing ball is $h(t) = 4\sin^2(8\pi t)$, where h is height in metres and t is time in seconds. When is the ball at a height of 2 m?

4) What is wrong with the following “solution”?

$$\sin x \tan x = \sin x$$

$$\cancel{\sin x} \tan x = \cancel{\sin x}$$

$$\tan x = 1$$

$$x = \frac{\pi}{4} \quad \text{or} \quad x = \frac{5\pi}{4}$$

5) Determine **all** possible solutions for $\sin^2 x = \frac{1}{2}$.

6) Determine **all** possible solutions for $\tan^2 x - \tan x = 0$.

7) Solve the following equation on the interval $0 \leq x \leq 2\pi$, if possible.

$$\sin^2 x + 3\cos^2 x = 0$$

Answers

1) a) $199.5^\circ, 340.5^\circ$ b) $36.9^\circ, 323.1^\circ$ c) $45^\circ, 90^\circ, 225^\circ, 270^\circ$ d) $47.1^\circ, 132.9^\circ$
e) $30^\circ, 90^\circ, 150^\circ, 270^\circ$ f) $48.6^\circ, 101.3^\circ, 131.4^\circ, 281.3^\circ$

2) a) $\frac{3\pi}{2}$ b) no solution c) $1.23, \frac{2\pi}{3}, \frac{4\pi}{3}, 5.05$ d) $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$
e) $\frac{\pi}{4}, 2.035, \frac{5\pi}{4}, 5.176$ f) $\frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{3\pi}{2}$

3) $t = \frac{1}{32}, \frac{3}{32}, \frac{5}{32}, \frac{7}{32}, \dots$ 5) $x = \frac{\pi}{4} + \frac{n\pi}{2}$, where n is an integer

6) $x = n\pi$ or $x = \frac{\pi}{4} + n\pi$, where n is an integer 7) no solution