NAME: SOLUTIONS

42

KNOW /10	APP /14	INQ /14	COMM /4

MCV4U1 - UNIT 3 – DERIVATIVES AND THEIR APPLICATIONS

TEST

ROUND ALL ANSWERS TO THE NEAREST TENTH, UNLESS OTHERWISE STATED.

1) The position, from a fixed point, of an object moving along a straight line is given by $s(t) = \frac{4}{3}t^3 - 10t^2 + 24t + 6$, where $s(t)$ is the object's position, in metres, after t seconds.

a) Find the object's velocity at 3 seconds. (*K – 2 marks*)

$s'(t) = 4t^2 - 20t + 24$

$s'(3) = 4(3)^2 - 20(3) + 24$

$= 0 \text{ m/s}$

b) Find the object's acceleration at 2.4 seconds. (*K – 2 marks*)

$s''(t) = 8t - 20$

$s''(2.4) = 8(2.4) - 20$

$= -0.8 \text{ m/s}^2$

c) Determine when the object is stationary. (*K – 2 marks*)

$s'(t) = 0$

$4t^2 - 20t + 24 = 0$

$t^2 - 5t + 6 = 0$

$(t-2)(t-3) = 0$

$t = 2, 3$

∴ stationary at 2 seconds and 3 seconds

d) Calculate the total distance travelled by the object during the first 5 seconds. (*K – 4 marks*)

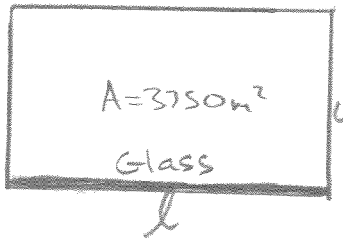
t	$s(t)$	
0	6	
2	$\frac{74}{3}$	$\frac{56}{3}$ m
3	24	$\frac{2}{3}$ m
5	$\frac{128}{3}$	$\frac{56}{3}$ m

Total distance $= \frac{56}{3} + \frac{2}{3} + \frac{56}{3}$

$= 38 \text{ m}$

2) A supermarket is designed to have a rectangular floor area of 3750 m^2, with three walls made of cement blocks and one wall made of glass. In order to conform to the building code, the length of the glass wall must not exceed 60 m, but must not be less than 30 m. The cost per linear metre of constructing a glass wall is twice that of constructing a cement wall. Find the dimensions of the floor area that will minimize the construction cost of building the walls. (*A – 7 marks*)

Let a be the cost of cement per metre



$lw = 3750$

$w = \frac{3750}{l}$

$C = a(2w + l) + 2al$

$= 2aw + 3al$

$= 2a\left(\frac{3750}{l}\right) + 3al$

$= 7500al^{-1} + 3al$

$C' = -\frac{7500a}{l^2} + 3a$

For min, $0 = -\frac{7500a}{l^2} + 3a$

$\frac{7500a}{l^2} = 3a$

$7500 = 3l^2$

$2500 = l^2$

$l = 50$

Check min,

$C'' = \frac{15000a}{l^3}$

$C''(50) > 0$

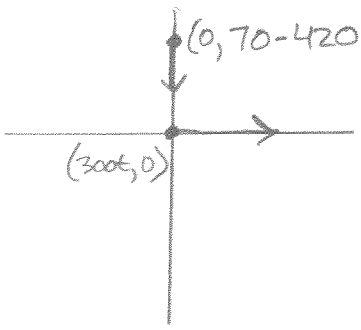
$\therefore$ min

$w = \frac{3750}{50}$

$= 75$

$\therefore$ length should be 50m and width should be 75m (glass wall is on length side)

3) At 11:00 on a beautiful Saturday morning, a plane flying east at 300 km/h passed over the Brantford airport. Ten minutes later, another plane flying south at 420 km/h passed over the same point. If the planes were flying at the same altitude, at what time were they closest to each other? **You do not need to check for a minimum in this question.** (*A – 7 marks*)



$d = \sqrt{(70-420t-0)^2 + (0-300t)^2}$

$= \sqrt{(70-420t)^2 + (300t)^2}$

Suffices to minimize $f(t) = (70-420t)^2 + (300t)^2$

$f'(t) = 2(70-420t)(-420) + 2(300t)(300)$

$= -58800 + 352800t + 180000t$

$= 532800t - 58800$

For min,

$0 = 532800t - 58800$

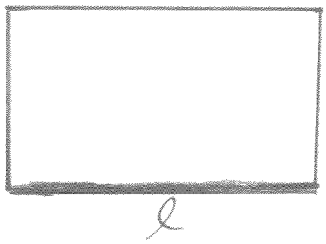
$t = 0.11$ hours

$= 6.6$ minutes

$= 6$ minutes and 36 seconds

$\therefore$ closest at 11:06:36

4) A rectangular lot is to be fenced using two different types of fencing. Three sides will be fenced with a standard fencing that costs \$7/m. The other side is next to a parking lot, so it requires heavier fencing that costs \$9/m. If the company has \$14 000 to spend on materials, what dimensions should be used to maximize the area enclosed by the fence? **You do not need to check for a maximum in this question.** (*I - 7 marks*)



$7(l+2w)+9l=14000$

$16l+14w=14000$

$w=\frac{14000-16l}{14}$

$w=1000-\frac{8}{7}l$

$A=lw$

$=l(1000-\frac{8}{7}l)$

$=1000l-\frac{8}{7}l^2$

$A'=1000-\frac{16}{7}l$

For max,

$0=1000-\frac{16}{7}l$

$\frac{16}{7}l=1000$

$16l=7000$

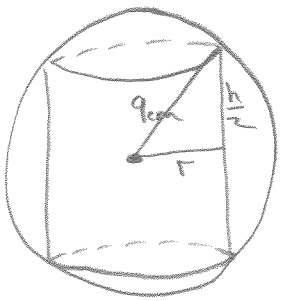
$l=437.5$

$\therefore w=1000-\frac{8}{7}(437.5)$

$=500$

$\therefore$ width should be 500m, length should be 437.5m (heavier fencing along length)

5) A cylinder is inscribed in a sphere that has a diameter of 18 cm. What are the dimensions of the cylinder that has the maximum volume? (*I – 7 marks*)



$\left(\frac{h}{2}\right)^2+r^2=9^2$

$\frac{h^2}{4}+r^2=81$

$r^2=81-\frac{h^2}{4}$

$V=\pi r^2h$

$=\pi(81-\frac{h^2}{4})h$

$=\pi(81h-\frac{h^3}{4})$

$V'=\pi(81-\frac{3}{4}h^2)$

For max,

$0=\pi(81-\frac{3}{4}h^2)$

$0=81-\frac{3}{4}h^2$

$\frac{3}{4}h^2=81$

$h^2=108$

$h=\pm 10.4$

Check for max:

$V''=\pi(-\frac{3}{2}h)$

$V''(10.4)<0$ $\therefore$ max

$\therefore r^2=81-\frac{10.4^2}{4}$

$r\doteq 7.3$

$\therefore$ height is 10.4cm and radius is 7.3cm

6) Find the absolute maximum and minimum values of $f(x)=\dfrac{4x}{x^2+1}$, on the domain $0\le x\le 4$.

(*C – 4 marks*)

$f(x)=\dfrac{4x}{x^2+1}$

$f'(x)=\dfrac{4(x^2+1)-2x(4x)}{(x^2+1)^2}$

$=\dfrac{4x^2+4-8x^2}{(x^2+1)^2}$

$=\dfrac{-4x^2+4}{(x^2+1)^2}$

$=\dfrac{-4(x^2-1)}{(x^2+1)^2}$

$=\dfrac{-4(x-1)(x+1)}{(x^2+1)^2}$

∴ critical numbers are $-1, 1$

~~$f(-1)=-2$~~ not in domain

$f(0)=0$

$f(1)=2$

$f(4)\doteq 0.9$

∴ absolute maximum value is 2.
absolute minimum value is 0.

7) **COMPLETE ON A SEPARATE PAGE**

A truck driver is planning to travel a distance of 2758 km. He will be driving 8 hours per day on Monday and Wednesday, and 6.5 hours on Tuesday. 98.5% of the distance will be traveled on the highway and the remaining 1.5% will be driven on city streets. He is paid \$18/h when he is on the road and \$12.37/h when he is not on the road (washroom breaks excluded). Last time the driver made this trip, he noticed that the price of fuel dropped by 1.2% for each 250 km closer to Halifax. The cost of gas in Hamilton is \$0.92/L. If the truck consumes fuel at a rate of $\left(0.005x+\dfrac{2}{x}\right)$ L/km, where x is its speed in km/h, and the driver has to replace a tire in Belleville, are horses pretty? (*0 marks – yes, this question is a joke and you are not required to answer it in any way*)