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NAME: ____________________

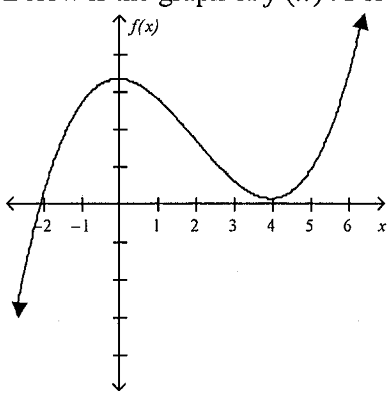
KNOW /12	APP (CURVE SKETCH)	INQ /12	COMM /6

MCV4U1 - UNIT 4 – CURVE SKETCHING
TEST

Multiple Choice

For questions 1 through 12, identify the choice that best answers the question. (K – 1 mark each)

c 1. Below is the graph of $f(x)$. For what value(s) of x does $f''(x) = 0$?



a. -2
b. 0 and 4
c. 2
d. 0, 2 and 4

c 2. For the graph of $f(x)$ in question #1, on what interval(s) is $f'(x) < 0$?

a. $x < -2$
b. $x < 0$ and $x > 4$
c. $0 < x < 4$
d. $x < 2$

a 3. Let $f'(x) = \dfrac{(x-8)(x+4)}{(x-6)^2}$. For what values of x is $f(x)$ increasing?

a. $x < -4$ and $x > 8$
b. $-4 < x < 6$
c. $6 < x < 8$
d. $f(x)$ is never increasing

d 4. For a function $f(x)$, the derivative is $f'(x) = \dfrac{x^2 + 2x - 15}{x^2}$. What are the critical numbers for $f(x)$?

a. only -5 and 3
b. only -3 and 5
c. only 0
d. -5, 3 and 0

a 5. For a function $f(x)$, if we know that $f'(c) > 0$, which of the following **must** be true?

a. $f(x)$ is increasing at $x = c$
b. $f(x)$ is decreasing at $x = c$
c. $f(x)$ is positive at $x = c$
d. $f(x)$ is concave up at $x = c$

d 6. Let $f(x) = \dfrac{2x^2 - 5x - 3}{x - 3}$. What types of asymptotes does $f(x)$ have?

a. Vertical and horizontal
b. Only vertical
c. Vertical and linear oblique
d. $f(x)$ does not have any asymptotes

b 7. Let $f(x)=\sqrt[3]{(x-7)^2}$. What is the concavity of $f(x)$ at $x=12$?

a. Concave up
b. Concave down
c. Neither concave up nor concave down
d. Unknown

a 8. If the graph of $f'(x)$ has a local maximum at $x=0$, what can we conclude about the polynomial function $f(x)$ at $x=0$?

a. There is an inflection point
b. There is a local maximum
c. $f(x)$ is positive
d. We cannot conclude anything

b 9. Let $f(x)=x^5-5x^4+3x+7$. For what value(s) of x does $f(x)$ have a point of inflection?

a. Only 0
b. Only 3
c. 0 and 3
d. Value(s) other than 0 and 3

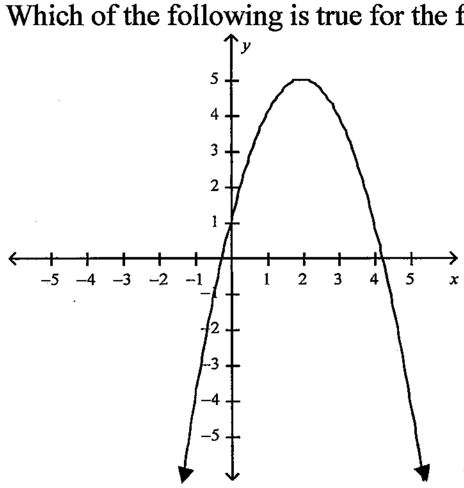
d 10. If $f'(5)=0$ and $f''(5)<-18$, what occurs on the graph of $f(x)$ at $x=5$?

a. An inflection point
b. A cusp
c. A local minimum
d. A local maximum

a 11. Consider the functions $f(x)=7x^3+5x^2+10x+5$ and $g(x)=7x^3+5x^2+10x+17$. How do the graphs of $f'(x)$ and $g'(x)$ compare?

a. $f'(x)$ and $g'(x)$ have identical graphs
b. $g'(x)$ has greater tangent slopes at any given x-value
c. $f'(x)$ is higher than $g'(x)$
d. $g'(x)$ is higher than $f'(x)$

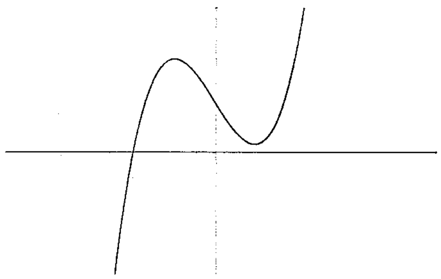
b 12. Which of the following is true for the function *f*(*x*) shown below when $x<2$?



a. $f'(x)>0$ and $f''(x)>0$
b. $f'(x)>0$ and $f''(x)<0$
c. $f'(x)<0$ and $f''(x)>0$
d. $f'(x)<0$ and $f''(x)<0$

13) For each of the following graphs on the left, write the letter of the corresponding derivative graph on the right. (*I – 6 marks*)

d



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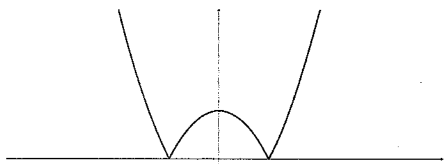

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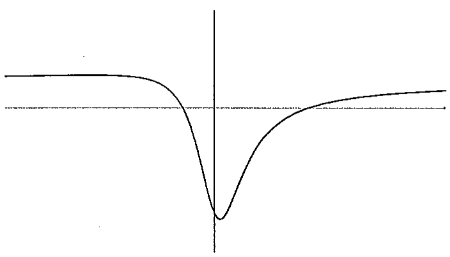
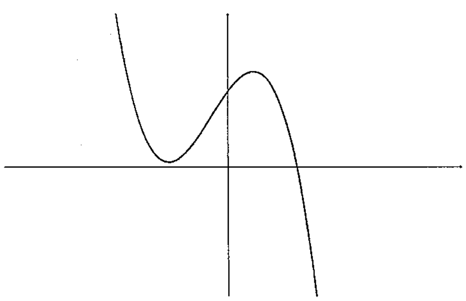
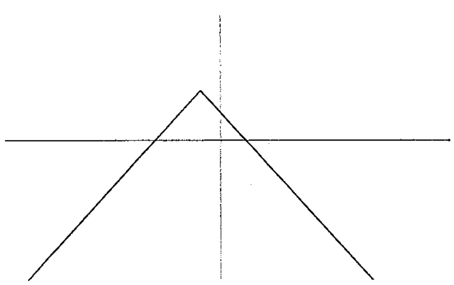


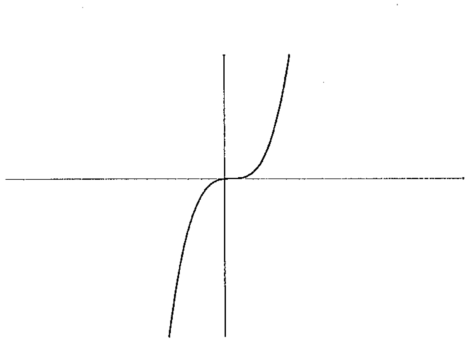
c



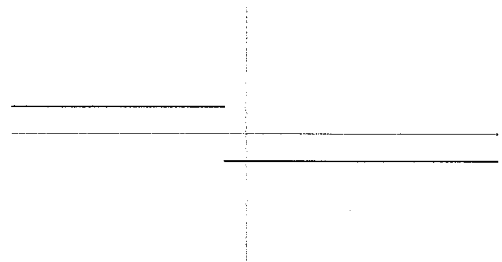
a



f



a)



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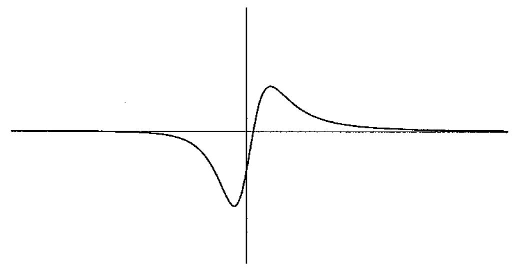
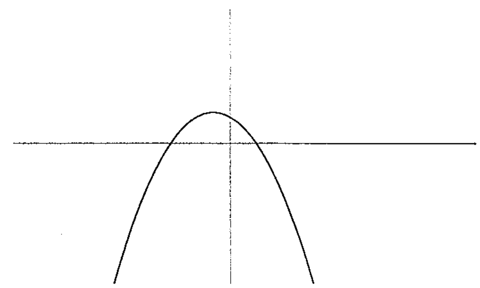
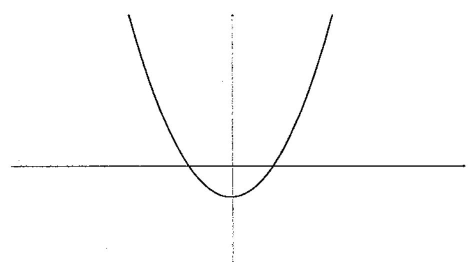


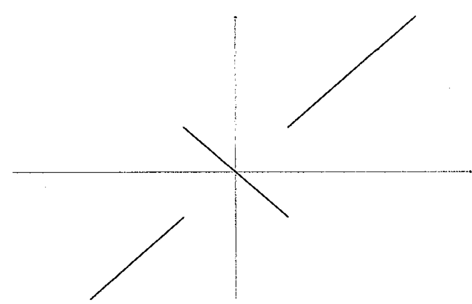
c)



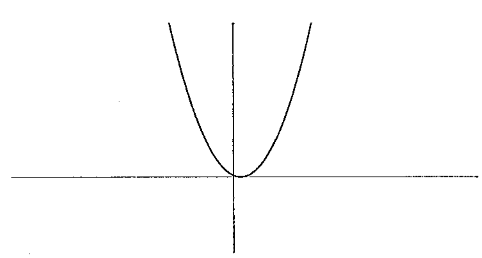
d)



e)



f)



14) The function $f(x) = ax^3 + bx^2 + cx + d$ has a local minimum at $(-2, -207)$ and an inflection point where $x = 4$. Furthermore, the slope of the tangent at $x = -1$ is 99. Determine the vales of *a*, *b*, *c* and *d*. (*I – 6 marks*)

$f'(x) = 3ax^2 + 2bx + c$

$f''(x) = 6ax + 2b$

$f''(4) = 0$

$6a(4) + 2b = 0$

$24a + 2b = 0$

① $12a + b = 0$

$f'(-2) = 0$

$3a(-2)^2 + 2b(-2) + c = 0$

② $12a - 4b + c = 0$

$f'(-1) = 99$

$3a(-1)^2 + 2b(-1) + c = 99$

③ $3a - 2b + c = 99$

② $12a - 4b + c = 0$

③ $3a - 2b + c = 99$

Subtract $9a - 2b = -99$ ④

$2\times$① $24a + 2b = 0$

④ $9a - 2b = -99$

Add $33a = -99$

$a = -3$

Sub. in ①

$12(-3) + b = 0$

$b = 36$

Sub. in ②

$12(-3) - 4(36) + c = 0$

$c = 180$

$f(-2) = -207$

$\therefore -3(-2)^3 + 36(-2)^2 + 180(-2) + d = -207$

$-192 + d = -207$

$d = -15$

$$\boxed{\begin{aligned}\therefore a &= -3\\ b &= 36\\ c &= 180\\ d &= -15\end{aligned}}$$

15) While investigating a function on the domain $\{x \in \Re \mid 0 \leq x \leq 6\}$, Tamara created the following tables.

x	0	3	5	6
$f(x)$	-1	4	-1	-3
$f'(x)$	5	0	-8	-10
$f''(x)$	-1	-3	0	3

x	$0 < x < 3$	$3 < x < 5$	$5 < x < 6$
$f'(x)$	+	–	–
$f''(x)$	–	–	+

Based on Tamara's observations, determine each of the following. (*C – 4 marks*)

a) The concavity of the function's graph at $x = 3$: Concave Up / Concave Down (*circle one*)

b) The graph's increasing/decreasing behaviour at $x = 5$: Increasing / Decreasing (*circle one*)

c) The coordinates of all inflection points (give the *x* and *y* value): $(5, -1)$

d) The coordinates of all local maximum and local minimum points (give the *x* and *y* value): $(3, 4)$ ← (local max)

16) During a deep conversation about the graphs of differentiable functions, Gary claimed that if $f''(7) = 0$ for a function $f(x)$, then there must be an inflection point at $x = 7$. Is Gary's claim correct? Explain. (*C – 2 marks*)

No. In order to have an inflection point, the concavity must switch from positive to negative or vice versa. It is possible to have $f''(7) = 0$ without a change in concavity at $x = 7$. For example, $f(x) = (x-7)^4$, any linear function or any constant function has $f''(7) = 0$, but no inflection point at $x = 7$.