

## Transformations of Cubic and Quartic Functions



Up to this point, you have transformed the graphs of many base functions, such as the following:

$$y = x \quad y = x^2 \quad y = \sqrt{x} \quad y = \frac{1}{x} \quad y = b^x \quad y = |x|$$

In this lesson we will consider transformations on the graphs of  $y = x^3$  and  $y = x^4$ .

How do you think the transformations on these functions will differ from the way we transformed previous functions?

# NOT AT ALL!



A couple of examples...

- Describe the transformations that must be applied to  $y = x^3$  to graph  $y = -3\left(\frac{1}{2}x+1\right)^3 - 3$  and then graph the function.

$$y = -3\left(\frac{1}{2}x+1\right)^3 - 3$$

$$y = -3\left[\frac{1}{2}(x+2)\right]^3 - 3$$

### Transformations

- vertical reflection (in  $x$ -axis)
- vertical stretch of factor 3
- horizontal stretch of factor 2
- horizontal shift 2 units left
- vertical shift 3 units down

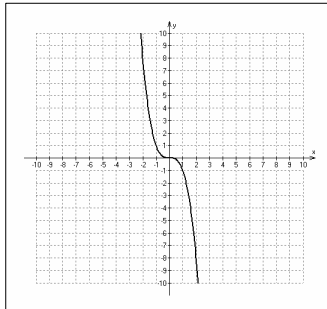
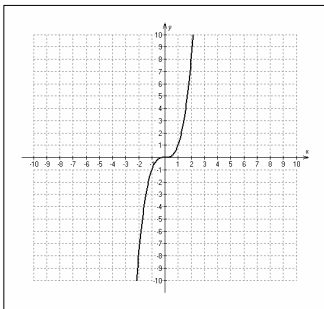
The graph...

$$y = -3\left[\frac{1}{2}(x+2)\right]^3 - 3$$

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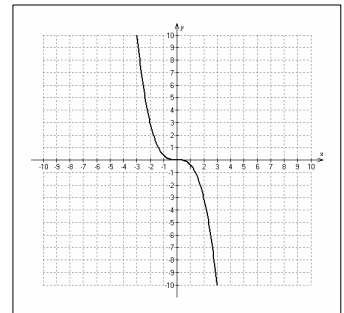
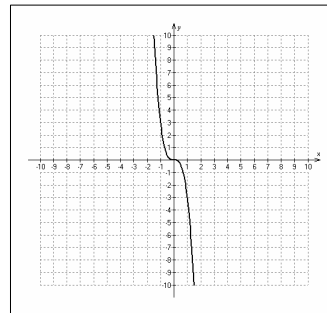
Parent function  $y = x^3$

Vertical reflection (in  $x$ -axis)



Vertical stretch of factor 3

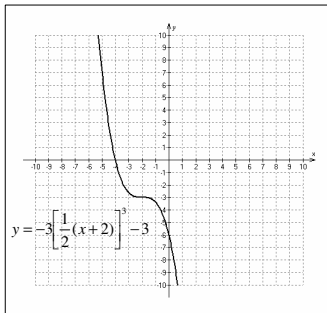
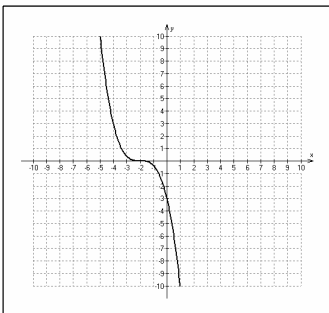
Horizontal stretch of factor 2



$$y = -3\left[\frac{1}{2}(x+2)\right]^3 - 3$$

Horizontal shift 2 units left

Vertical shift 3 units down



NOTE: You may want to combine several transformations in each step!

- Determine the  $x$ -intercept(s) of the function  $y = 3(x+6)^4 - 48$ .

$$y = 3(x+6)^4 - 48$$

$$0 = 3(x+6)^4 - 48$$

$$48 = 3(x+6)^4$$

$$\frac{48}{3} = (x+6)^4$$

$$16 = (x+6)^4$$

$$\pm\sqrt[4]{16} = x+6$$

$$\pm 2 = x+6$$

$$-6 \pm 2 = x$$

Therefore,

$$x = -6 + 2 \quad \text{or} \quad x = -6 - 2$$

$$x = -4 \quad \text{or} \quad x = -8$$

So, the  $x$ -intercepts are -4 and -8.