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EXPLORATION Investigating Factors of Polynomial Functions

PURPOSE

In this exploration, you will look for patterns when dividing with polynomial functions. You will use these patterns to help predict the remainder.

EQUIPMENT

- scientific calculator or graphing calculator

PROCEDURE

What is the relation between a binomial divisor and the remainder?

1. **(a)** Let $P(x) = x^2 + 6x - 3$.

(b) Evaluate $P(2)$.

(c) Divide $P(x)$ by $(x - 2)$. Use either long division or synthetic division.

(d) What is the relation between $x = 2$ and the divisor $x - 2$?

(e) Compare the remainder from (c) with $P(2)$. What do you notice?

2. **(a)** Let $P(x) = 4x^2 - 8x + 2$.

(b) Evaluate $P(-1)$.

(c) Use either long division or synthetic division to divide $P(x)$ by $(x + 1)$.

(d) What relation exists between $x = -1$ and the divisor $x + 1$?

(e) Compare the remainder from (c) with $P(-1)$. What do you notice?

3. **(a)** Let $P(x) = -2x^3 + 3x^2 - 9x - 5$.

(b) Evaluate $P(4)$.

(c) Use either long or synthetic division to divide $P(x)$ by $(x - 4)$.

(d) What relation exists between $x = 4$ and the divisor $x - 4$?

(e) Compare the remainder from (c) with $P(4)$. What do you notice?

4. **(a)** Let $P(x) = 6x^4 - 2x^3 + 4x^2 - 5x + 1$. Predict the remainder if $P(x)$ is divided by $x + 3$.

(b) Use either long division or synthetic division to verify your prediction.

ANALYSIS

DEVELOPING A NOTION OF THE REMAINDER THEOREM

1. Suppose you must divide any polynomial $P(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_2x^2 + a_1x + a_0$ by a binomial $x - k$. How can you determine the remainder without actually dividing?
2. **(a)** Let $P(x) = 8x^3 - 4x^2 + 6x - 10$.
   (b) Evaluate $P\left(\frac{1}{2}\right)$.
   (c) Use either long division or synthetic division to divide $P(x)$ by $(2x - 1)$.
   (d) What is the relationship between $x = \frac{1}{2}$ and the divisor $2x - 1$?
   (e) Compare the remainder from (c) with $P\left(\frac{1}{2}\right)$. What do you notice?
3. Suppose you divide any polynomial $P(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_2x^2 + a_1x + a_0$ by a binomial $jx - k$. How can you determine the remainder without actually dividing?

FURTHER YOUR ANALYSIS

DEVELOPING A NOTION OF THE FACTOR THEOREM

1. When you divide a polynomial by a binomial, what must be the remainder if the binomial is a factor of the polynomial?
2. **(a)** Predict the remainder if $x^2 - 7x - 18$ is divided by $x + 2$.
   (b) Use long division or synthetic division to check your prediction.
   (c) Is $x + 2$ a factor of $x^2 - 7x - 18$? Explain your answer.
3. Determine which binomials are factors of $x^3 - x^2 - 14x + 24$ without dividing.
   (a) $x + 2$ **(b)** $x + 1$ **(c)** $x - 3$ **(d)** $x - 2$
4. Let a polynomial be $P(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_2x^2 + a_1x + a_0$ and a binomial be $x - k$. How can you determine if the binomial is a factor of $P(x)$?
5. Suggest a method for deciding whether a binomial is a factor of polynomial $P(x)$ or not.
6. Determine at least one binomial factor for each polynomial.
   (a) $P(x) = x^3 - 4x^2 + x + 6$ **(b)** $P(x) = 3x^3 + 4x^2 - 5x - 2$