CHARACTERISTICS OF POLYNOMIAL FUNCTIONS

END BEHAVIOR, ZEROS AND TURNING POINTS

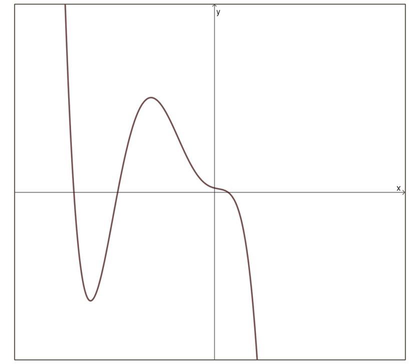
End Behaviour for Polynomial Functions

Warm-Up

State the end behaviour for the function shown in the graph on the right.

As $x \to \infty, y \to -\infty$

As $x \to -\infty, y \to \infty$



Question for Discussion

Given the equation of a **polynomial function**, can we determine the function's end behaviour?

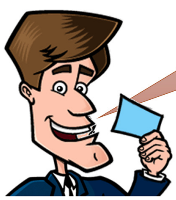
YES!

It turns out that in order to determine a polynomial function's end behaviour, all we need to know is its **degree** and **leading coefficient**.

Highest exponent on x

$$f(x) = 4x^3 - 7x^2 + 5x - 11$$

Coefficient of the highest power of x



Name that degree and leading coefficient!

$$f(x) = -6x^5 + 3x^3 - x + 2$$

Degree: 5
Leading Coefficient: -6

$$f(x) = 5x^3 + x^4 + 9x - 8$$

Degree: 4
Leading Coefficient: 1

$$f(x) = 2.1x^2 + 3.8x^4 - 5.2x^6$$

Degree: 6
Leading Coefficient: -5.2

Back to End Behaviour

Consider the equation of the polynomial function shown below

Now, think about what happens to each term as x approaches infinity.

GETS REALLY BIG, REALLY FAST

GETS A LOT BIGGER

GETS BIGGER

Doesn't change

$$f(x) = 2x^3 + 4x^2 + 5x + 8$$

As x approaches infinity, the highest power of x quickly "overpowers" the other terms, making them seem insignificant.

So, if we want to know what our function does as x approaches infinity, we just need to consider what happens to its highest power of x as x approaches infinity.

Now, as x approaches infinity, $2x^3$ approaches **infinity**.

We can use the same approach to determine the behaviour of $f(x)$ as x approaches negative infinity.

As x approaches negative infinity, $2x^3$ approaches **negative infinity**.

$$f(x) = 2x^3 + 4x^2 + 5x + 8$$

As x approaches infinity, $2x^3$ approaches **infinity**.

As x approaches negative infinity, $2x^3$ approaches **negative infinity**.

Therefore, for our function $f(x) = 2x^3 + 4x^2 + 5x + 8$, the end behaviour is:

As $x \to \infty,\ f(x) \to \infty$
As $x \to -\infty,\ f(x) \to -\infty$

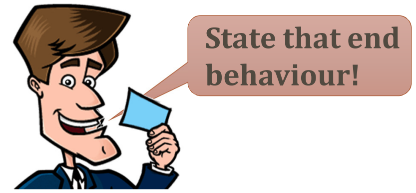
WARNING

The idea that only the highest power of x is significant applies only when consider very large positive or very large negative values of x. For smaller values of x each term can have a significant impact on the function's value.

Questions for Discussion

NO — Would the end behaviour for the function $g(x) = -2x^3 + 4x^2 + 5x + 8$ be the same as above?

NO — Would the end behaviour for the function $h(x) = 2x^4 + 4x^2 + 5x + 8$ be the same as above?



Function	End Behaviour
$f(x) = -6x^4 + 3x^3 - x + 2$ **Degree:** 4 (even degree) **Leading Coefficient:** -6 (negative leading coefficient)	As $x \to \infty,\ f(x) \to -\infty$ As $x \to -\infty,\ f(x) \to -\infty$
$y = 4x^2 - 5x^3 - x + 2$ **Degree:** 3 (odd degree) **Leading Coefficient:** -5 (negative leading coefficient)	As $x \to \infty,\ f(x) \to -\infty$ As $x \to -\infty,\ f(x) \to \infty$

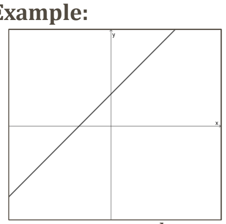
Zeros and Turning Points

How many zeros can a polynomial function have?

Degree 1 Polynomial

Name: Linear

Example:



$y = mx + b$

Given a linear equation in the form $y = mx + b$, where $m \neq 0$, how could we determine where the graph intersects the x-axis?

- Set $y = 0$ and solve for x.

How many answers will we get? 1

How many turning points could a linear function have? 0

Summary

Ends of graph: Opposite directions
Maximum number of zeros: 1
Minimum number of zeros: 1
Maximum number of turning points: 0
Minimum number of turning points: 0

Either
As $x \to \infty,\ y \to \infty$
As $x \to -\infty,\ y \to -\infty$
or
As $x \to \infty,\ y \to -\infty$
As $x \to -\infty,\ y \to \infty$

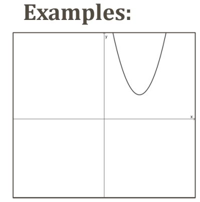
Degree 2 Polynomial

Name: Quadratic

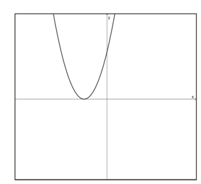
How many zeros could a quadratic function have? 0, 1 or 2

For $y = ax^2 + bx + c$, if we set $y = 0$ and solve for x (by factoring, completing the square or quadratic formula) we could get 0, 1 or 2 real answers.

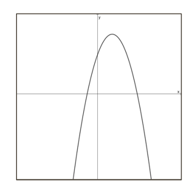
Examples:



$y = x^2 - 6x + 13$



$y = (x + 2)^2$



$y = -3(x + 1)(x - 4)$

Notice that in order to have two zeros, there must be a turning point!

Summary

Ends of graph: Same direction
Maximum number of zeros: 2
Minimum number of zeros: 0
Maximum number of turning points: 1
Minimum number of turning points: 1

Either
As $x \to \infty,\ y \to \infty$
As $x \to -\infty,\ y \to \infty$
or
As $x \to \infty,\ y \to -\infty$
As $x \to -\infty,\ y \to -\infty$

Degree 3 Polynomial

Name: Cubic

How many zeros could a cubic function have?

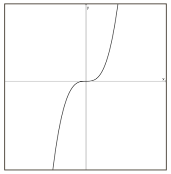
1, 2 or 3

For $y = ax^3 + bx^2 + cx + d$, if we set $y = 0$ we might be able to factor and get up to 3 answers.

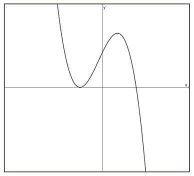
Q: Why must a cubic function have at least one zero?

A: The ends of the graph go opposite ways.

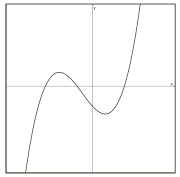
Examples:



$y = x^3$



$y = -(x-3)(x+2)^2$



$y = (x-2)(x+1)(x+3)$

Notice that in order to have three zeros, there must be two turning points!

Summary

Ends of graph:	Opposite directions
Maximum number of zeros:	3
Minimum number of zeros:	1
Maximum number of turning points:	2
Minimum number of turning points:	0

Degree 55 Polynomial

Name: ????????

Summary

Ends of graph:	Opposite directions
Maximum number of zeros:	55
Minimum number of zeros:	1
Maximum number of turning points:	54
Minimum number of turning points:	0

Degree 56 Polynomial

Name: ????????

Summary

Ends of graph:	Same direction
Maximum number of zeros:	56
Minimum number of zeros:	0
Maximum number of turning points:	55
Minimum number of turning points:	1

Big Ideas for Polynomial Functions

- The maximum number of zeros is the same as the degree.
- The maximum number of turning points is one less than the degree.
- Odd degree functions must have at least one zero.
- Even degree functions don't need to have any zeros.
- Odd degree functions don't need to have any turning points.
- Even degree functions must have at least one turning point.