Function: a rule that states for every output there is one input: x can only have one y

Vertical Line Test: if any vertical line passes through more than one point on the graph then it is NOT a function.

Function Notation:

$$f(x) = \pm a(\pm b(x-h)) \pm K$$

f is a function of x

"f@x" NOT f times x

Domain: the set of INPUTS in a relation (all of the x-values)

Range: the set of OUTPUTS in a relation (all of the y-values)

Absolute Val Notation

1 x 1

Writing Absolute value as a piecewise function:

$$\rightarrow f(x) = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

Intervals of Increasing: when the graph rises from left to right. $f(x_1) < f(x_2)$ whenever $x_1 < x_2$

Base Function: refers to the original function before a transformations have been appl

Intervals of Decreasing: when the graph falls from left to righ $f(x_1) > f(x_2)$ whenever $x_1 < x_2$

bsolute Maximum: highest point on the graph

↳ $f(a) \geq f(x)$ for all of x in the domain of f.

bsolute Minimum: lowest point on the graph

↳ $f(b) \leq f(x)$ for all of x in the domain of f.

Local Maximum: when a function changes from increasing to decreasi

Local Minimum: when a function changes from decreasing to increasin

Note: MAX. or MIN. values that occur @ endpoints are NOT considere to be local Maximums or local Minim

UNIT 1 - Functions

ontinuity: • a continuous function oes not contain any holes OR reaks over its entire domain.

• a discontinuous function does contain holes & breaks which an occur anywhere through domain

Symmetry: • an even function Symetric about the y-axis

• an odd function has rotational Symetry about the origin.

lgebraically Solving Symmetry:

$f(-x) = f(x)$

• replace all "x" with "-x" still result in the original f(x) function.

Even Symmetry

$f(-x) = -f(x)$

↳ replace all "x" with "-x" & result in opposite signs of the original f(x) function

Odd Symmetry

End Behaviour:

• refers to what happens as the function x-valu become very large positively OR very large negatively. Otherwise known as the functio x-values approach infinity + or as the function x-values approach infinity −

End Behaviour Notation:

As $x \rightarrow \infty$, $f(x) \rightarrow$

As $x \rightarrow -\infty$, $f(x) \rightarrow$

ransformation Equation:

$$f(x) = \pm a(\pm b(x \pm h)) \pm K$$

: vertical stretch OR compression

: horizontal stretch OR compression

: horizontal shift right OR left

: vertical shift up OR down.

±: infront of a/b = reflection

±: infront of h = left/right

±: infront of K: up/down

Inverse Relations: the inverse of a function (f) is the function that "UNDOES" f.

Inverse of a function Notation:

↳ f^{-1}

ecewise Function:

function defined by using les on different intervals.

• Consists of several pieces all f which are functions.

iecewise Notation:

$\rightarrow f(x) = \{ \quad \}$

Exploring Operations with Functions:

↳ when adding two functions we add the two output values for every valid input

↳ We can only add where their domains overlap

↳ The same ideas apply to subtraction and multiplication.

Unit 2 - Rates of Change

istance: the total distance travelled by an object. NEVER a negative value.

isplacement: an objects position elative to a fixed point can be a negative value.

peed: how fast an object is travelling without regard to direction. NEVER a negative value.

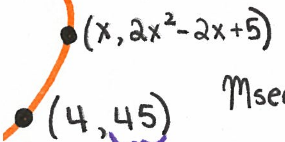
verage Rate of Change: calculates he amount of change in one item divided y the corresponding amount of change in another . change in distance over change in time

↳ $A.R.O.C = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$ = Slope

ethod to find I.R.O.C:

e create a formula for the slope the secant line through the point interest & a general point on the curve

e.) $f(x) = 3x^2 - 2x + 5$; where $x = 4$



$$m_{sec} = \frac{\Delta f(x)}{\Delta x}$$

$$= \frac{3x^2 - 2x + 5 - 45}{x - 4}$$

$$= \frac{3x^2 - 2x - 40}{x - 4}$$

$$= \frac{(3x-10)\cancel{(x-4)}}{\cancel{x-4}}$$

Will ALWAYS factor & cancel. You do not need to do that though.

$$= 3x - 10,\ x \neq 4$$

ound by bbing in 4 to f(x).

m left:

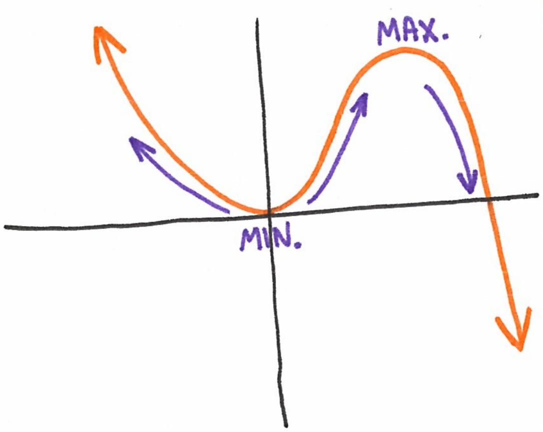
	Msec
	21.7
	21.97
	21.997
	21.9997

∴ we see the secant slopes approach a value of 22.

om Right:

	Msec
.1	22.3
01	22.03
001	22.003
001	22.0003

∴ tangent slopes @ x=4 is approxi 22.



Velocity: how fast an object is travelling and the direction of its motion can be a negative value.

Displacement-Time Graph: graphically displays a describes an objects position relative to a fixed poi

Velocity-Time Graph: graphically displays & desc how fast an object is travelling and its direction. Fou by finding the slopes of a displacement-time graph.

NOTE: velocity equals zero on the MAX. & MIN. on a displaceme time graph | when the displacement-time graph is a straight line the velocity-time graph will be a horizontal line to represent a constant speed | when the displacement-tim graph is a horizontal line the velocity-time graph wi show a velocity of zero | positive slopes on the displacem time graph means positive values on velocity-time graph & the same thing with negative slopes |

Instantaneous Rate of Change: the rate of change at a particular moment I.R.O.C of a function at a given point is the slope of the tangent line at that point.

The Tangent Line: touches exactly one point of the graph

Slope of a Tangent: the slope of tangent at P is the limiting value o the slopes of the secants PQ as Q approaches point P.

Note: If we want the rate of change of a function at a particular point we are reall after the slope of the tangent line @ that poi cannot find the slope of the tangent must use 2 methods to find it, which gives us the I.R.O.C.

MAXimums & MINimum values: the slope of the tangent where a local MAX & MIN occurs is zero.

Determining Maximums & Minimums:

★ We can determine minimums when the slope of the secant goes from a negative value, from the left side of a specific point, to a positive secant slope, from the right side. These values are also small #'s

★ We can determine maximums when the slope of the secant goes from a positive value to a negative secant value. These values are also large numbers.

UNIT 3 - Polynomial Functions

Polynomial Functions: an expression that nly has powers in it. It must be whole #'s. ach of these powers will have a coefficient hat can be any number you can think of.
.e: $5x^7 - 3x^2 + 7$ $3x^2 + 7y^2 + 1$

The Degree: the value of the highest exponent of the variable.

Turning Point: a point on a curve that is highe or lower than all nearby points.

Zeros of Polynomial Functions in Factored Form:

the zero of a linear function f(x) is s in $f(x) = K(x-s)$
the zeros of a quadratic function f(x) is one s and t in $f(x) = K(x-s)(x-t)$
the zeros of a cubic function f(x) are s, t and u in $f(x) = K(x-s)(x-t)(x-u)$

NOTE: if you want the zeros factor the polynomial & set each factor to zero.

Characteristics of Polynomial Functions:

→ A polynomial of degree n has n-1 possible turning points
.e. 55 factors = 54 possible turning points

→ **Order:** the exponent to which a factor in an algebraic expression is raised.
ie: $f(x) = (x-3)^2(x-1)$ order of $(x-3)$ is 2 the order of $(x-1)$ is 1.

→ **Family:** a set of polynomial functions whose equations have the same degree & whose graphs have common characteristics

Dividing Polynomials:

→ Method 1: long division

ind $(x^2+7x-3) \div (x+1)$

$$\begin{array}{r l}
 & x + 6 \\
x+1 \,\big) & x^2 + 7x - 3 \\
 - & x^2 + x \\
 & 6x - 3 \\
 - & 6x + 6 \\
 & -9
\end{array}$$

$x^2+7x-3 = (x+1)(x+6)-9$
dividend = divisor • quotient + remainder

ITE: e remainder st be a aller degree than e quoitent degree zero lower than degree one.

→ Method 2: Synthetic Division

Steps:
1) decide on the K-value
2) List the coefficients in the dividend
3) Bring down the first coefficient
4) Multiply & add to get the other coefficier
5) Write out the results using the variale power:

Find $(4x^3 - 5x^2 + 3x - 7) \div (x-2)$
↳ K value = 2

2	4	-5	3	-7
	↓	8	6	18
	4	3	9	11
	x^2	x	constant	remaind

$4x^2 + 3x + 9$ R11
↳ $(x-2)(4x^2+3x+9) + 11$

Factoring Polynomials:

he Remainder Theorem:

ien a polynomial P(x) is divided by x-K e remainder is equal to P(K)
hen a polynomial P(x) is divided by jx-K remainder is equal to $P(\frac{K}{j})$

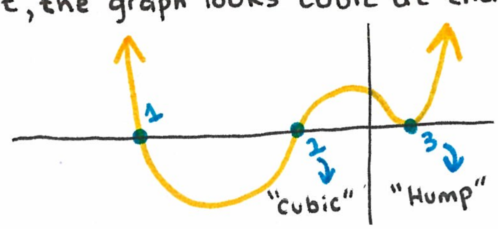
The Factor Theorem:

x-K is a factor of P(x) ONLY IF P(K) = 0
jx-K is a factor of P(x) ONLY IF $P(\frac{K}{j}) = 0$

STEPS:
1) Find a K value such that P(K) = 0
2) divide p(x) by (x-K) to find the other
3) Factor
4) Repeat the process until you get all the the factors down to a degree 2.
5) factor any degree 2 factors.

Graphing Polynomials:

TEPS: → Factor the polynomials if they e not already factored.
→ Each factor is where the graph is zero so ot the points (the zeros)
→ If the bracket has an even number on it, e graph makes a "hump" at that zero
→ If the factor bracket has an odd number it, the graph looks cubic at that zero.



Factoring a Sum OR Difference of Cubes:

$$A^3 - B^3 = (A-B)(A^2 + AB + B^2)$$
$$A^3 + B^3 = (A+B)(A^2 - AB + B^2)$$

Leading Coefficient: the term of highest power of x.

Steps to Solving Polynomial Functions:
1.) Substitute in the value required for the function
2.) Move all terms to one side so that one side equals zero
3.) Factor & solve for the values in the requested domain

Solve for the x ... $P(x)=10$ in $P(x)=x^4+x^3-16x^2+26x-2$

Set $P(x)=10$

$\to 10=x^4-x^3-16x^2+26x-2$

$0=x^4-x^3-16x^2+26x-2-10$

$p(x)=x^4+x^3-16^2+26x-12 \to P(1)=0$

$\therefore (x-1)$ is a factor $\to k=1$

1 | 1	1	-16	26	-12
↓	1	2	-14	12
1	2	-14	12	☺

$\therefore x^4+x^3-16x^2+26x-2=(x-1)(x^3+2x^2-14x$ ...

$\hookrightarrow$ CONTINUE UNTIL ALL FACTORS ARE FOUND.

Solving Linear Inequalities:

→ A linear inequality is an inequality that contains an algebraic expression of degree 1.

$<$ = less than, $>$ = greater than.

Solve linear inequalites similar to the way we solve linear equations★

Equation:

$3x-1=8$

$3x=8+1$

$3x=9$

$x=9/3$

$x=3$

Inequality:

$3x-1<8$

$3x<8+1$

$3x<9$

$x<9/3$

$x<3$

Set notation: $\{x\in R \mid x<3\}$

Interval notation: $x\in(-\infty,3)$

$x-29<4x-5<x+12$

$x-29<4x-5$

$-24<3x$

$-8<x$

$4x-5<x+12$

$3x<17$

$x<17/3$

$=-8<x<17/3$

SOLVE $x^4-8x<0$

$x(x^3-8)<0$

$x(x-2)(x^2+2x+4)<0$

We want x values where $x(x-2)(x^2+2x+4)=0$

$x=0, x=2$ OR $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$=\frac{-2\pm\sqrt{-12}}{2}$

= no solution.

∴ NO ZEROS

$x<0$	$0<x<2$	$x>2$
+	−	+

$x^4-8x<0$ when $0<x<2$

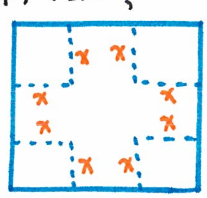
UNIT 4 - Polynomial Equations & Inequalities

Rates of Changes in Polynomial Functions:

The process of A.R.O.C & I.R.O.C is the same as dealing with inequalities.

WORD PROBLEMS:

→ A piece of metal 65cm × 33cm has congruent squares notched out of each corner to make so the metal can be folded up into an open top box. If the box is to have a volume of 4779 cm³, find the length, width, & height of all possible boxes.



33-2x

65-2x

$4779=x(33-2x)(65-2x)$

$4779=x(2145-66x-130x+4x^2)$

$4779=x(4x^2-196x+2145)$

$0=4x^3-196x^2+2145x-4779$

Let $f(x)=4x^3-196x^2+2145x-4779$

$f(x)=0$

3 | 4	-196	2145	-4779	
↓	12	-552	4779	
4	-184	1593	☺	

$x=\frac{184\pm\sqrt{184^2-4(4)(1593)}}{2(4)}$

$x=34.43459663$

$x=11.56540337$

To Find x-values that work:

$65-2(3)=59$ ✓

$33-2(3)=27$

$65-2(34.4)=-3.8$ ✗

$33-2(34.4)=-35.8$

$65-2(11.6)=41.8$ ✓

$33-2(11.6)=9.8$

∴ Possibilities of the dimensions are 59cm by 27cm & 9.8cm by 11.6cm.

UNIT 5 - Rational Functions, Equations & Inequalities

Reciprocal Functions:

↳ $y = \frac{1}{dog}$ ★dog = function

Steps to graphing reciprocal functions:

- → state domain and its restrictions
- → state intercepts
- → state asymptotes (horizontal, vertical, oblique)
- → make a positive/negative interval chart (includes zeros & vertical asymptotes)

★don't invite amy in ★

Oblique Asymptote: occurs the polynomial on the numerator is one degree higher than the polynomial on the denominator.

e: $y = \frac{2x^3 - x^2 + 3}{x^2}$ ∴ Oblique asymptope is $y = 2x - 1$

$y = 2x - 1 + \frac{2}{x^2}$

Rational Function: In the form of

$h(x) = \frac{f(x)}{g(x)}$ where $f(x)$ & $g(x)$ are polynomials

NOTE: $g(x) \neq 0$

Vertical Asymptote: the same as domain restrictions where the graph does not exist.

Horizontal Asymptote: occurs in 3 different scenarios

$\frac{\text{lower degree}}{\text{higher degree}}$ = horizontal asymptote of $y = 0$

$\frac{\text{higher degree}}{\text{lower degree}}$ = NO horizontal asymptote

$\frac{\text{Same degree}}{\text{same degree}}$ = divide coefficients to get horizontal asymptote

★NOTE: an oblique and horizontal asymptote cannot occur at the same time

SOLVING RATIONAL EQUATIONS:

→ When solving rational equations with multiple fractions either clear the fraction or get a common denominator.

owe $\frac{x+3}{x-4} = \frac{x-1}{x+2}$, $x \neq 4, \neq -2$

$\frac{\cancel{(x-4)}(x+2)}{1} \cdot \frac{x+3}{\cancel{x-4}} = \frac{(x-4)\cancel{(x+2)}}{1} \cdot \frac{x-1}{\cancel{x+2}}$

$(x+2)(x+3) = (x-4)(x-1)$

$x^2 + 5x + 6 = x^2 - 5x + 4$

$10x + 2 = 0$

$10x = -2$

$x = -1/5$

Check

L.S	R.S
$\frac{\frac{-1}{5} + 3}{\frac{-1}{5} - 4} \times \frac{5}{5}$	$\frac{\frac{-1}{5} - 1}{\frac{-1}{5} + 2} \times \frac{5}{5}$
$= \frac{-1 + 15}{-1 - 20}$	$= \frac{-1 - 5}{-1 + 10}$
$= \frac{-14}{21}$	$= \frac{-6}{9}$
$= -\frac{2}{3}$	$= \frac{-2}{3}$

∴ LS = RS

SOLVING RATIONAL INEQUALITIES:

↳ Solve $\frac{7}{x-3} \geq \frac{2}{x+4}$

↳ We use the same thing: "Don't invite amy in" to solve.

↳ $\frac{7}{x-3} - \frac{2}{x+4} \geq 0$

$\frac{7(x+4) - 2(x-3)}{(x-3)(x+4)} \geq 0$

$\frac{5x+34}{(x-3)(x-4)} \geq 0$

Domain: $x \neq 3$, $x \neq -4$

Intercepts:

x-int:

$0 = \frac{5x+34}{(x-3)(x+4)}$

$0 = 5x + 34$

$\frac{-34}{5} = x$

$x = -6.8$

y-int:

$y = \frac{5(0)+34}{(0-3)(0+4)}$

$= \frac{34}{12}$

$y = 2.8$

Asymptotes:

- $x = 3$
- $x = -4$
- $x = 0$

Positive/negative Intervals:

	$x < -6.8$	$-6.8 < x < -4$	$-4 < x < 3$	$x >$
$\frac{5(x+6.8)}{(x-3)(x+4)}$	−	+	−	+

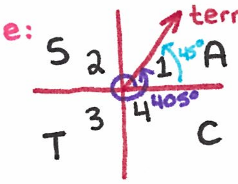
∴ $\frac{7}{x-3} - \frac{2}{x+4} \geq 0$... when $-6.8 \leq x < -4$ & $x > 3$

RATES OF CHANGE OF RATIONAL FUNCTIONS

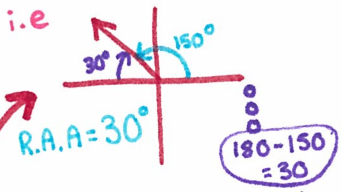
↳ the same methods of I.R.O.C is used when dealing with rational functions.

Coterminal Angles: when 2 angles in standard position have the same teminal arm

i.e:

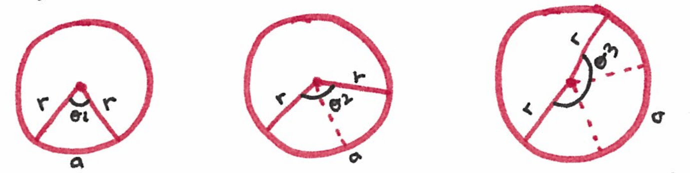


i.e



Relate Acute Angle: an angle in standard position that is between the terminal arm & the x-axis. (between 0° and 90°)

Radian: the measure of any angle ... subtended @ the centre of a circle by an arc equal in length to the radius of the circle.



$\theta_1 = 1$ rad $\theta_2 = 2$ rad $\theta_3 = 3$ rad

Radian Formula:

Number of radians = $\dfrac{\text{arc length}}{\text{radius}}$

OR $\theta = \dfrac{a}{r}$

Converting Between Degrees and Radians:

→ To convert between degrees and radians you must know that 1 radian = 180° otherwise written as π rad = 180°.

i.e → $\dfrac{\pi}{3}\cancel{\text{rad}} \times \dfrac{180°}{\cancel{\pi\text{rad}}} = \dfrac{180°}{3} = 60°$

→ $45\cancel{°} \times \dfrac{\pi \text{rad}}{180\cancel{°}} = \dfrac{45\pi \text{rad}}{180} = \dfrac{\pi}{4}$ rad

→ π rad $= \dfrac{180°}{\pi}$ & $1° = \dfrac{\pi}{180}$ rad

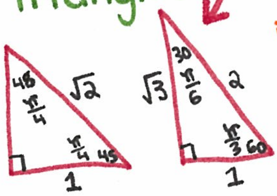
NOTE: if the number does not have a degree symbol then assume its rad 1 revolution 2π rad.

Reciprocal Trigonomic Ratios:

$\csc\theta = \dfrac{1}{\sin\theta}$ $\sec\theta = \dfrac{1}{\cos\theta}$ $\cot\theta = \dfrac{1}{\tan\theta}$

CAST Rule: →

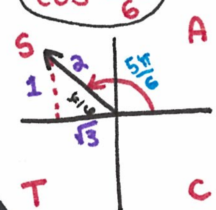
Special Triangles:



UNIT 6 - Trigonometric Functions

Using Special Triangles:

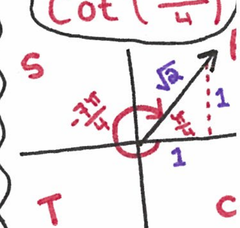
$\cos\frac{5\pi}{6}$



R.A.A = $\frac{\pi}{6}$

$\cos\frac{\pi}{6} = -\frac{\sqrt{3}}{2}$

$\cot\left(\frac{-7\pi}{4}\right)$



R.A.A = $\frac{\pi}{4}$

$\cot\left(\frac{\pi}{4}\right) = 1$

Properties of:	Domain:	Range:	Period	MAX.	MIN.	y-int.	x-intercepts.	Amp.	Asymptotes.
$y = \sin x$	$x \in R$ (degrees+radians)	$-1 \le y \le 1$ (degrees+radians)	360° OR 2π rad	1	-1	0	$180°n, n\in Z$ OR $\pi n, n \in Z$	1	none
$y = \cos x$	$x \in R$ (degrees+radians)	$-1 \le y \le 1$ (degrees+radians)	360° OR 2π rad	1	-1	1	$90° + 180°n, n \in Z$ OR $\frac{\pi}{2} + \pi n, n \in Z$	1	None
$y = \tan x$	$x \in R \mid x \ne 90 + 180n, n \in Z$ ↳ Degrees $x \in R \mid x \ne \frac{\pi}{2} + \pi n, n \in Z$ ↳ Radians	$y \in \mathbb{R}$ (degrees+Radians)	π	BOTH local & absolute: None	None	0 (degrees +radians)	$180n, n \in Z$ OR $\pi n, n \in Z$		$90 + 180n, n \in Z$ OR $\frac{\pi}{2} + \pi n, n \in Z$
$y = \csc x$	$x \in R \mid x \ne K\pi, K \in Z$	$y \in R \mid y \le -1$ & $y \ge 1$	2π	Local: 1 Absolute: none	-1	None	None		$x = K\pi, K \in Z$
$y = \sec x$	$x \in R \mid x \ne K\frac{\pi}{2}, K \in Z$	$y \in R \mid y \le -1$ & y 1	2π	local: 1 Absolute: none	-1	1	None		$x = K\frac{\pi}{2}, K \in Z$
$y = \cot x$	$x \in R \mid x \ne K\pi, K \in Z$	$y \in \mathbb{R}$	π	local: none Absolute: none		0	$K\frac{\pi}{2}, K \in Z$		$x = K\pi, K \in Z$

RATES of change of trigonometric Functions:

→ the process of average rate of change and istantaneous rate of change is the same when concerning trigonometric functions.

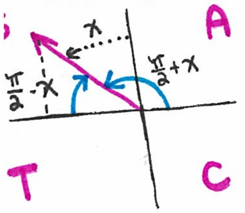
UNIT 7 - Trigonometric Identities & Equations

Equivalent Trigonometric Equations

- $\sin(-x) = -\sin x$ ↳ $\cos(-x) = \cos x$ ↳ $\tan(-x) = -\tan x$
- $\sin(\frac{\pi}{2} - x) = \cos x$ ↳ $\cos(\frac{\pi}{2} - x) = \sin x$ ↳ $\tan(\frac{\pi}{2} - x) = \cot x$

Express as a trigonometric Function:

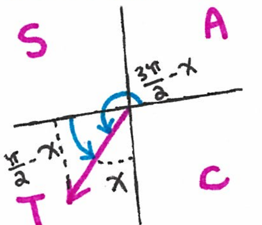
→ $\tan(\frac{\pi}{2} + x)$



R.A.A $= \frac{\pi}{2} - x$

→ $\tan(\frac{\pi}{2} - x) = -\cot x$

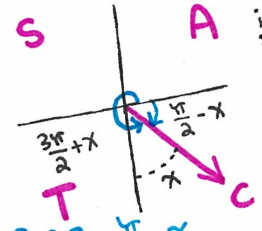
↳ $\sin(\frac{3\pi}{2} - x)$



R.A.A $= \frac{\pi}{2} - x$

↳ $\sin(\frac{\pi}{2} - x) = -\cos x$

↳ $\cos(\frac{3\pi}{2} + x)$



R.A.A: $\frac{\pi}{2} - x$

↳ $\cos(\frac{\pi}{2} - x) = -\sin x$

Steps to solving trig. functions

1.) Draw the angle
2.) Find the related acute angle
3.) Express the ratio given in terms of the related acute angle.
4.) Watch ~~cr~~ sign change

EXAMPLE: Prove $\frac{\sin 2x}{1 - \cos 2x} = 2\csc 2x - \tan x$

L.S

↳ $\frac{\sin 2x}{1 - \cos 2x}$

$= \frac{2\sin x \cos x}{1 - (\cos^2 x - \sin^2 x)}$

$= \frac{\cancel{2}\sin x \cos x}{\cancel{2}\sin^{\cancel{2}} x}$

$= \boxed{\frac{\cos x}{\sin x}}$

R.S

↳ $2\csc 2x - \tan x$

$= 2\frac{1}{\sin 2x} - \frac{\sin x}{\cos x}$

$= \cancel{2}\frac{1}{\cancel{2}\sin x \cos x} - \frac{\sin x}{\cos x}$

$= \frac{1}{\sin x \cos x} - \frac{\sin x}{\sin x \cos x}$

$= \frac{1 - \sin^2 x}{\sin x \cos x}$

$= \frac{\cos^{\cancel{2}} x}{\sin x \cancel{\cos x}}$

$\boxed{= \frac{\cos x}{\sin x}}$

LS = RS
∴ TRUE

Steps to solve linear trig. eq:

★ use special triangles or draw their graphs

1.) Find the R.A.A
2.) Use the negative to determine which quad. the terminal arm could be in
3.) Draw the terminal arm & R.A.A (sketch it)
4.) using domain state the values of x that fit that criteria

TRIGONOMETRIC TOOL BOX:

Reciprocal Identities:

- $\csc x = \frac{1}{\sin x}$
- $\sec x = \frac{1}{\cos x}$
- $\cot x = \frac{1}{\tan x}$

Quotient Identities:

- $\tan x = \frac{\sin x}{\cos x}$
- $\cot x = \frac{\cos x}{\sin x}$

Pythagorean Identities:

- $\sin^2 x + \cos^2 x = 1$
- $\sec^2 x = 1 + \tan^2 x$
- $\csc^2 x = 1 + \cot^2 x$

Addition & Subtraction Formulas:

$\sin(x+y) = \sin x \cos y + \cos x \sin y$

$\sin(x-y) = \sin x \cos y - \cos x \sin y$

$\cos(x+y) = \cos x \cos y - \sin x \sin y$

$\cos(x-y) = \cos x \cos y + \sin x \sin y$

$\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$

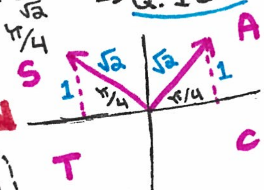
$\tan(x-y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$

Solve the following equations for $0° \le \theta \le 360°$ $(0 \le \theta \le 2\pi)$

↳ $\boxed{\sqrt{2}\sin\theta - 1 = 0}$

$\sin\theta = \frac{1}{\sqrt{2}}$ ↳ Q.1 & 2

↳ R.A.A $= \pi/4$



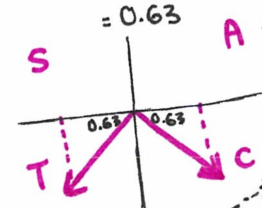
$\theta = \pi/4$, $\theta = \pi - \pi/4 = 3\pi/4$

↳ $\boxed{\csc\theta = -1.7013}$

$\frac{1}{\sin}\theta = -1.7013$

$\sin\theta = \frac{1}{1.7013}$ (Q.3 & 4)

↳ R.A.A $= \sin^{-1}(\frac{1}{1.7013})$
$= 0.63$

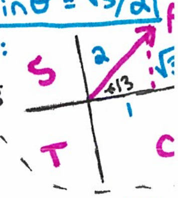


$\theta = \pi + 0.63°$
$= 3.77$ rad

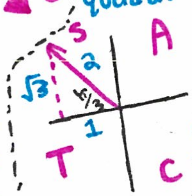
$\theta = 2\pi - 0.63°$
$= 5.65$ rad.

↳ $\boxed{\sin\theta = \sqrt{3}/2}$

quad. 1:
R.A.A $= \frac{\pi}{3}$



quad 2:



R.A.A $= \frac{\pi}{3}$

$\therefore x = \pi/3$

$x = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$

Double Angle Formulas:

- $\sin 2x = 2\sin x \cos x$
- $\cos 2x = \cos^2 x - \sin^2 x$
  $= 2\cos^2 x - 1$
  $= 1 - 2\sin^2 x$
- $\tan 2x = \frac{2\tan x}{1 - \tan^2 x}$

UNIT 8- Logarithms

Logarithmic Function:
→ Where the input is the base & the output is the exponent.

Equation of a log:
↳ $\log_a x$ to exponent that must be applied to the base (a) to obtain the value of x. (the log function)

Exponential Form:
→ $x = a^y$

Exponential form of $y = \log_3 x$
↳ $3^y = x$
$x = 3^y$

Inverse of $f(x) = 4^x$
→ $y = 4^x$
For inverse:
$x = 4^y$
$y = \log_4 x$
• Inverse $= f^{-1}(x) = \log_4 x$

Common Logarithms:
↳ $\log x = \log_{10} x$

Graphing Logarithmic Functions
Steps:
1.) Change the function from logarithmic to exponential
2.) Graph the exponential function
3.) Apply all the transformations to the graph.

Important Properties of Logarithms:
↳ $\log_a 1 = 0 \quad \log_a a^x = x \quad a^{\log_a x} = x$

Log Laws:
there are 3 laws; logarithm product law, logarithm quotient law, logarithm power of a power log.

① $\log_a mn = \log_a m + \log_a n$
② $\log_a\left(\frac{m}{n}\right) = \log_a m - \log_a n$
③ $\log_a m^y = y \log_a m$

Evaluating Logarithms with a base other than 10:
→ $\log_a x = \frac{\log x}{\log a}$

pH Scale:
→ $pH = -\log[H+]$

Richter Scale:
you subtract the magnitude then you put that number ten to that number
i.e = $7 - 5.5 = 1.5 \rightarrow 10^{1.5} = 31.6$

Sound Intensity:
You subtract the decibles then divide that number by 10 & finally set that number to the power of 10.
i.e = $125 - 53 = 72 \rightarrow 72 \div 10 = 7.2 \rightarrow 10^{7.2} = 15.8$ million.

Compound Intrest Formula:
↳ Concerns about money problems

$A = P(1 + i)^n$

- A: Final amount
- P: Principal (initial)
- i: 1 + interest rate per period
- n: number of compounds

quarterly: 4
monthly: 12
weekly: 52
daily: 365

Exponential Models:
any situation that involves repeated multiplication can be modelled using:

$y = c(a^x)$

- y: Final
- c: Initial
- a: Growth factor

Doubling: 2
Half Life: 1/2
Growth by 12%: 1.12
Decay by 25%: 0.75

$(3)^x = 2430$	$3^{x+2} - 3^x = 216$	$3^x = 100$
$3^x = \frac{2430}{10}$	↳ $3^x(3^2 - 1) = 216$	$\log 3^x = \log 100$
$3^x = 243$	$3^x(8) = 216$	$x \log 3 = \log 100$
$3^x = 3^5$	$3^x = 27$	$x = \frac{\log 100}{\log 3}$
$\therefore x = 5$	$3^x = 3^3$	$\therefore x = 4.192$
	$\therefore x = 3$	

Ex. Mr. Todd needs \$5000. He invests \$3000 at 18% compounded semi-annually. How long will it take to become \$5000.

$A = P(1+i)^n$

$5000 = 3000\left(1 + \frac{0.18}{2}\right)^n$

$\frac{5000}{3000} = (1 + 1.09)^n$

$\frac{5}{3} = 1.09^n$

$\log \frac{5}{3} = \log 1.09^n$

$\log \frac{5}{3} = n \log 1.09$

$\frac{\log \frac{5}{3}}{\log 1.09} = n$

$n = 5.93$ semi-annually.

$\frac{5.93}{2} = 2.96$ years

$\therefore 3$ years