Recall:

The Fundamental Theorem of Calculus, Part 1

If f is continuous on $[a, b]$, then the function

$$F(x) = \int_a^x f(t)\,dt$$

has a derivative at every point x in $[a, b]$, and

$$\frac{dF}{dx} = \frac{d}{dx}\int_a^x f(t)\,dt = f(x).$$

Examples

1) For each of the following, determine $\dfrac{dy}{dx}$.

a) $y = \int_7^x (3t^2 + 5)dt$

Note: The lower limit of integration and the dummy variable of integration do not affect the derivative here.

b) $y = \int_{15}^x (3r^2 + 5)dr$

Note:

$$\int_a^b f(x)dx = -\int_b^a f(x)dx$$

c) $y = \int_x^{12} (3t^2 + 5)dt$

d) $y = \int_{-3}^{x^3} \cos t\,dt$

e) $y = \int_{7}^{4x^2} \left(2t^3 + 12t^2 + 3\right) dt$

f) $y = \int_{x}^{5} 3t \sin t dt$

g) $y = \int_{2x}^{x^2} \frac{1}{2 + e^t} dt$

h) $y = \int_{x}^{x^3} \ln(3t + 5) dt$

2) Determine a function $y = f(x)$ with derivative $\frac{dy}{dx} = \tan x$ that satisfies the condition $f(3) = 5$.

Recall:

> ### The Fundamental Theorem of Calculus, Part 2
>
> If f is continuous at every point of $[a, b]$, and if F is any antiderivative of f on $[a, b]$, then
>
> $$\int_a^b f(x)\,dx = F(b) - F(a).$$
>
> This part of the Fundamental Theorem is also called the **Integral Evaluation Theorem.**

> **Integral Evaluation Notation**
>
> The usual notation for $F(b) - F(a)$ is
>
> $$F(x)\Big]_a^b \quad \text{or} \quad \Big[F(x)\Big]_a^b,$$
>
> depending on whether F has one or more terms. This notation provides a compact "recipe" for the evaluation, allowing us to show the antiderivative in an intermediate step.

Examples

Evaluate each of the following integrals.

a) $\displaystyle\int_3^6 (3x^2 + 5)dx$

b) $\displaystyle\int_0^5 24(3x+5)^3 dx$

c) $\displaystyle\int_{-\pi}^{\pi} 3\cos x dx$

d) $\displaystyle\int_0^5 \left(\frac{18x}{3x^2+5}\right)dx$