Sketch the graph of the function $f(x) = \frac{(x+1)^2}{x^2}$.

Domain

$\{x \in \Re \mid x \neq 0\}$

Intercepts

x-intercept:

$0 = \frac{(x+1)^2}{x^2}$

$0 = (x+1)^2$

$x = -1$

y-intercept:

$f(0) = \frac{(0+1)^2}{0^2}$

$= \frac{1}{0}$

undefined

$\therefore$ no y-intercept

Asymptotes

Vertical: $x = 0$
Horizontal: $y = 1$
Oblique: none

Intervals of Increase/Decrease

$$f(x) = \frac{(x+1)^2}{x^2}$$

$$f'(x) = \frac{2(x+1)x^2 - 2x(x+1)^2}{x^4}$$

$$= \frac{2x(x+1)[x-(x+1)]}{x^4}$$

$$= \frac{-2x(x+1)}{x^4}$$

$$= \frac{-2(x+1)}{x^3}$$

For critical numbers,

$$0 = \frac{-2(x+1)}{x^3}$$

x = -1

Also, $f'(x)$ is undefined for x = 0.

	x < -1	-1 < x < 0	x > 0
Sign of $f'(x)$	-	+	-
Increase/Decrease for $f(x)$	↘	↗	↘

Maximum and Minimum Points

Minimum at (-1, 0).

Concavity

$$f'(x) = \frac{-2(x+1)}{x^3}$$

$$f''(x) = \frac{-2x^3 - 3x^2(-2)(x+1)}{x^6}$$

$$= \frac{-2x^3 + 6x^2(x+1)}{x^6}$$

$$= \frac{-2x^2[x - 3(x+1)]}{x^6}$$

$$= \frac{-2x^2(-2x-3)}{x^6}$$

$$= \frac{2(2x+3)}{x^4}$$

For possible inflection points,

$$0 = \frac{2(2x+3)}{x^4}$$

$x = -1.5$

Also, $f''(x)$ is undefined for $x = 0$.

	$x < -1.5$	$-1.5 < x < 0$	$x > 0$
Sign of $f''(x)$	-	+	+
Concavity of $f(x)$	Down	Up	Up

Inflection Points

Inflection point at (-1.5, 0.11)