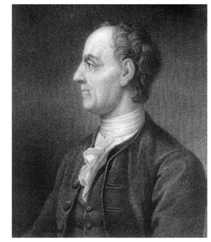
The Derivatives of Exponential Functions

We will begin our investigation of the derivatives of exponential functions by considering a very special exponential function:



$$f(x) = e^x$$

To find the derivative of this function, we will use our definition of the derivative.

Here we go!

$$f(x) = e^x$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{e^{x+h} - e^x}{h}$$

$$= \lim_{h \to 0} \frac{e^x\left(e^h - 1\right)}{h}$$

$$= e^x \lim_{h \to 0} \frac{e^h - 1}{h}$$

$$= e^x \lim_{h \to 0} \frac{e^h - 1}{h}$$

To find $\lim_{h \to 0} \frac{e^h - 1}{h}$, we can use tables.

h	$\frac{e^h - 1}{h}$
0.1	1.051709181
0.01	1.005016708
0.001	1.000500167
0.0001	1.000050002

h	$\frac{e^h - 1}{h}$
-0.1	0.951625819
-0.01	0.995016625
-0.001	0.999500166
-0.0001	0.999950001

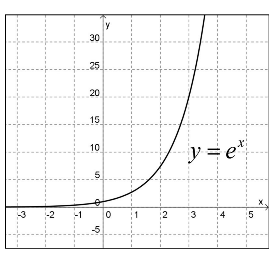
It appears that $\lim_{h \to 0} \frac{e^h - 1}{h} = 1.$

So, we have

$$f'(x) = e^x \lim_{h \to 0} \frac{e^h - 1}{h}$$

$$f'(x) = e^x (1)$$

$$f'(x) = e^x$$



So, the derivative of e^x is e^x!

In other words, at any point on the graph of $y = e^x$, the tangent slope is the same as the *y*-value!

Question for Discussion

Are there any other functions that are their own derivative?

$$\frac{d}{dx} e^x = e^x$$

Some examples (complete on a separate page):

Differentiate each of the following functions (with respect to *x*).

a) $f(x) = e^{2x}$ b) $f(x) = e^{x^2}$ c) $f(x) = xe^x$

d) $f(x) = x^4 e^{-x}$ e) $f(x) = \frac{x^3}{e^x}$ f) $f(x) = \frac{e^{x^2}}{3x^3 + 4x}$