TRIGONOMETRIC IDENTITIES
SUPPLEMENTARY PROBLEMS

The following identities involve the reciprocal, quotient, and Pythagorean relationships. Prove each one.

1. $\sin x \tan x = \sec x - \cos x$

2. $\cos^4 x - \sin^4 x = 1 - 2 \sin^2 x$

3. $\csc^2 x + \sec^2 x = \csc^2 x \sec^2 x$

4. $\cos^2 x \cos^2 y + \sin^2 x \sin^2 y + \sin^2 x \cos^2 y + \sin^2 y \cos^2 x = 1$

5. $\sec^2 x - \sec^2 y = \tan^2 x - \tan^2 y$

6. $\dfrac{\tan x + \tan y}{\cot x + \cot y} = (\tan x)(\tan y)$

7. $(\sec x - \cos x)(\csc x - \sin x) = \dfrac{\tan x}{1 + \tan^2 x}$

8. $\cos^6 x + \sin^6 x = 1 - 3 \sin^2 x + 3 \sin^4 x$

9. $\sec^6 x - \tan^6 x = 1 + 3 \tan^2 x \sec^2 x$

The following involve the addition and subtraction formulas.

10. $1 + \cot x \tan y = \dfrac{\sin(x + y)}{\sin x \cos y}$

11. $\cos(x + y)\cos y + \sin(x + y)\sin y = \cos x$

12. $\sin x - \tan y \cos x = \dfrac{\sin(x - y)}{\cos y}$

13. $\cos\left(\dfrac{3\pi}{4} + x\right) + \sin\left(\dfrac{3\pi}{4} - x\right) = 0$

14. $\dfrac{\tan\left(\dfrac{\pi}{4} + x\right) - \tan\left(\dfrac{\pi}{4} - x\right)}{\tan\left(\dfrac{\pi}{4} + x\right) + \tan\left(\dfrac{\pi}{4} - x\right)} = 2 \sin x \cos x$

15. $\sin(x + y)\sin(x - y) = \cos^2 y - \cos^2 x$

16. $\tan(x + y)\tan(x - y) = \dfrac{\sin^2 x - \sin^2 y}{\cos^2 x - \sin^2 y}$

17. $\dfrac{\tan(x - y) + \tan y}{1 - \tan(x - y)\tan y} = \tan x$

18. $\sin 5x = \sin x\,(\cos^2 2x - \sin^2 2x) + 2 \cos x \cos 2x \sin 2x$

The following involve related and co-related angles.

19. $\sin\left(\dfrac{\pi}{2} - x\right)\cot\left(\dfrac{\pi}{2} + x\right) = -\sin x$

20. $\cos(-x) + \cos(\pi - x) = \cos(\pi + x) + \cos x$

21. $\dfrac{\sin(\pi - x)}{\tan(\pi + x)} \dfrac{\cot\left(\dfrac{\pi}{2} - x\right)}{\tan\left(\dfrac{\pi}{2} + x\right)} \dfrac{\cos(2\pi - x)}{\sin(-x)} = \sin x$

22. $\dfrac{\sin(-x)}{\sin(\pi + x)} - \dfrac{\tan\left(\dfrac{\pi}{2} + x\right)}{\cot x} + \dfrac{\cos x}{\sin\left(\dfrac{\pi}{2} + x\right)} = 3$

23. $\dfrac{\csc(\pi - x)}{\sec(\pi + x)} \dfrac{\cos(-x)}{\cos\left(\dfrac{\pi}{2} + x\right)} = \cot^2 x$

24. $\dfrac{\cos\left(\dfrac{\pi}{2} + x\right)\sec(-x)\tan(\pi - x)}{\sec(2\pi + x)\sin(\pi + x)\cot\left(\dfrac{\pi}{2} - x\right)} = -1$

25. $\dfrac{\sin(\pi - x)\cos(\pi + x)\tan(2\pi - x)}{\sec\left(\dfrac{\pi}{2} + x\right)\csc\left(\dfrac{3\pi}{2} - x\right)\cot\left(\dfrac{3\pi}{2} + x\right)} = \sin^4 x - \sin^2 x$

The following involve the double angle formulas.

26. $\dfrac{\sin 2x}{1 + \cos 2x} = \tan x$

27. $\dfrac{1 + \cos x}{\sin x} = \cot \dfrac{x}{2}$

28. $2 \csc 2x = \sec x \csc x$

29. $2 \cot 2x = \cot x - \tan x$

30. $\dfrac{\cos 2x}{1 + \sin 2x} = \tan\left(\dfrac{\pi}{4} - x\right)$

31. $\dfrac{\cos x - \sin x}{\cos x + \sin x} = \sec 2x - \tan 2x$

32. $\dfrac{1 - \cos 2x + \sin 2x}{1 + \cos 2x + \sin 2x} = \tan x$

33. $\cos^6 x - \sin^6 x = \cos 2x\left(1 - \tfrac{1}{4} \sin^2 2x\right)$

34. $4(\cos^6 x + \sin^6 x) = 1 + 3 \cos^2 2x$

35. $\sec x - \tan x = \tan\left(\dfrac{\pi}{4} - \dfrac{x}{2}\right)$

36. $\dfrac{\sin 2x}{1 + \cos 2x} \dfrac{\cos x}{1 + \cos x} = \tan \dfrac{x}{2}$

The following involve a variety of formulas and identities.

37. $\sin^2 x + \cos^4 x = \cos^2 x + \sin^4 x$

38. $\tan x - \cot x = (\tan x - 1)(\cot x + 1)$

39. $\cos x = \sin x \tan^2 x \cot^3 x$

40. $(\sin x + \cos x)(\tan x + \cot x) = \sec x + \csc x$

41. $\sin^4 x + \cos^4 x = \sin^2 x(\csc^2 x - 2\cos^2 x)$

42. $\sin^3 x + \cos^3 x = (1 - \sin x \cos x)(\sin x + \cos x)$

43. $\cos\left(\dfrac{\pi}{12} - x\right)\sec\dfrac{\pi}{12} - \sin\left(\dfrac{\pi}{12} - x\right)\csc\dfrac{\pi}{12} = 4\sin x$

44. $\tan(x - y) + \tan(y - z) = \dfrac{\sec^2 y\,(\tan x - \tan z)}{(1 + \tan x \tan y)(1 + \tan y \tan z)}$

45. $\sin 8x = 8\sin x \cos x \cos 2x \cos 4x$

46. $\sin x = 1 - 2\sin^2\left(\dfrac{\pi}{4} - \dfrac{x}{2}\right)$

47. $\sin(x + y) + \sin(x - y) = 2\sin x \cos y$

48. $\dfrac{\sin(x - y)}{\sin x \sin y} + \dfrac{\sin(y - z)}{\sin y \sin z} + \dfrac{\sin(z - x)}{\sin z \sin x} = 0$

49. $\tan x + \tan(\pi - x) + \cot\left(\dfrac{\pi}{2} + x\right) = \tan(2\pi - x)$

50. $\sin\left(\dfrac{\pi}{2} + x\right)\cos(\pi - x)\cot\left(\dfrac{3\pi}{2} + x\right)$

$= \sin\left(\dfrac{\pi}{2} - x\right)\sin\left(\dfrac{3\pi}{2} - x\right)\cot\left(\dfrac{\pi}{2} + x\right)$

51. $\tan\left(\dfrac{\pi}{2} - x\right) - \cot\left(\dfrac{3\pi}{2} - x\right) + \tan(2\pi - x) - \cot(\pi - x)$

$= \dfrac{4 - 2\sec^2 x}{\tan x}$

52. $\tan(x + y + z) = \dfrac{\tan x + \tan y + \tan z - \tan x \tan y \tan z}{1 - \tan x \tan y - \tan x \tan z - \tan y \tan z}$

53. $\csc^2\left(\dfrac{\pi}{2} - x\right) = 1 + \sin^2 x \csc^2\left(\dfrac{\pi}{2} - x\right)$

54. $\tan\left(\dfrac{\pi}{4} + x\right) + \tan\left(\dfrac{\pi}{4} - x\right) = 2\sec 2x$

55. $\dfrac{1 - \sin 2x}{\cos 2x} = \dfrac{\cos 2x}{1 + \sin 2x}$

56. $\dfrac{\sin 4x}{1 - \cos 4x} \times \dfrac{1 - \cos 2x}{\cos 2x} = \tan x$