Higher Order Derivatives, Velocity and Acceleration

The Second Derivative

The second derivative of a function is simply the derivative of its derivative!

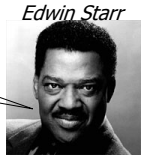
Example

Determine the second derivative of $f(x) = \dfrac{x}{1+x}$.

Common notation for the second derivative:

$$f''(x) \qquad y'' \qquad \frac{d^2y}{dx^2}$$

What is the second derivative good for?



Similar to the first derivative, the second derivative is useful for solving problems involving rates of change, especially those that deal with motion.

Problems Involving Motion

Suppose an object's displacement from a fixed point is described by the function $s(t)$.

What does $s'(t)$ represent?

Velocity, which is the rate of change of displacement. We often use $v(t)$ to denote velocity.

What does $s''(t)$ represent?

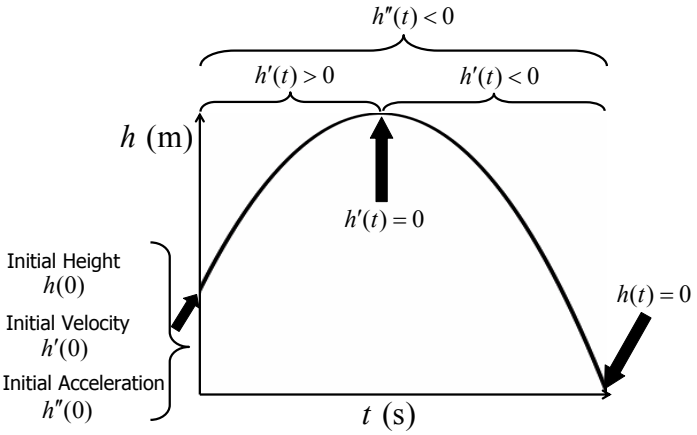
Acceleration, which is the rate of change of velocity. We often use $a(t)$ to denote acceleration.

Discussion

How would the motion of the object affect the signs of $s(t)$, $v(t)$ and $a(t)$?

Analyzing Motion

An thrown object's height, $h(t)$, is shown in the following graph. If the positive direction is upwards, analyze the key points on the graph.



Some examples…

Example 1

Analyzing the Motion of a Falling Object: Vertical Motion

A rock is tossed from a bridge 15 m above the water. The height of the rock, h, in metres above the water at t seconds can be modelled by the function $h(t) = -4.9t^2 + 12t + 15$.

(a) Determine the instantaneous velocity at 1 s and at 2 s.

(b) What is the velocity of the rock when it enters the water?

(c) Determine the initial velocity of the rock.

(d) When is the rock at its maximum height? What is the maximum height?

Example 2

Analyzing the Motion of a Moving Object: Horizontal Motion

The position of an object moving along a straight line can be modelled by the function $s(t) = 3t^3 - 40.5t^2 + 162t$, where s is the position in metres at t seconds and $t \geq 0$.

(a) Determine the initial position of the object.

(b) Determine the velocity at 2 s and 5 s.

(c) When is the object stationary?

(d) When is the object advancing? retreating?

(e) Determine the total distance travelled during the first eight seconds of motion.

Example 3

Acceleration and Horizontal Motion

The position at t seconds of a particle moving along a straight line is given by $s(t) = 3t^3 - 40.5t^2 + 162t$, where s is measured in metres and $t \geq 0$.

(a) Determine the acceleration at 6 s.

(b) Determine when the velocity is decreasing.

(c) Determine when the velocity is increasing.

(d) Determine when the velocity is not changing.