Assigned Work

1. Determine the points at which $f'(x) = 0$ for each of the following functions:

   a. $f(x) = x^3 + 6x^2 + 1$

   b. $f(x) = \sqrt{x^2 + 4}$

   c. $f(x) = (2x - 1)^2(x^2 - 9)$

   d. $f(x) = \dfrac{5x}{x^2 + 1}$

2. Explain how you would determine when a function is increasing or decreasing.

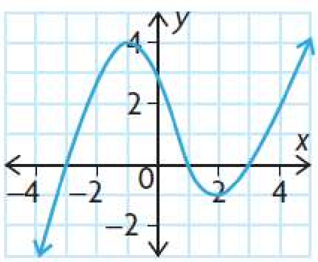
3. For each of the following graphs, state

   i. the intervals where the function is increasing

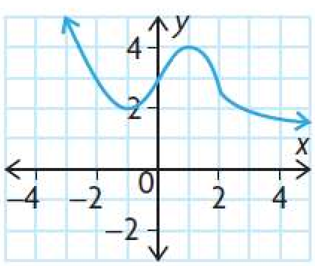
   ii. the intervals where the function is decreasing

   iii. the points where the tangent to the function is horizontal

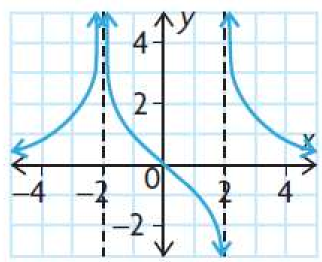
a.



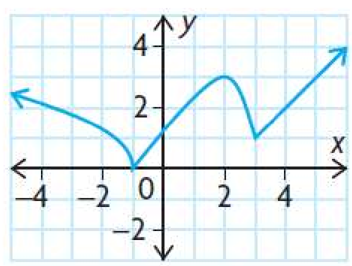
b.



c.



d.



4. Use a calculator to graph each of the following functions. Inspect the graph to estimate where the function is increasing and where it is decreasing. Verify your estimates with algebraic solutions.

   a. $f(x) = x^3 + 3x^2 + 1$

   b. $f(x) = x^5 - 5x^4 + 100$

   c. $f(x) = x + \dfrac{1}{x}$

   d. $f(x) = \dfrac{x - 1}{x^2 + 3}$

   e. $f(x) = 3x^4 + 4x^3 - 12x^2$

   f. $f(x) = x^4 + x^2 - 1$

5. Suppose that f is a differentiable function with the derivative $f'(x) = (x - 1)(x + 2)(x + 3)$. Determine the values of x for which the function f is increasing and the values of x for which the function is decreasing.

6. Sketch a graph of a function that is differentiable on the interval $-2 \leq x \leq 5$ and that satisfies the following conditions:
   - The graph of f passes through the points $(-1, 0)$ and $(2, 5)$.
   - The function f is decreasing on $-2 < x < -1$, increasing on $-1 < x < 2$, and decreasing again on $2 < x < 5$.

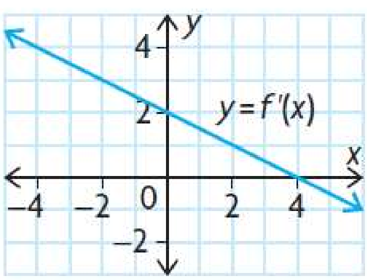
7. Find constants a, b, and c such that the graph of $f(x) = x^3 + ax^2 + bx + c$ will increase to the point $(-3, 18)$, decrease to the point $(1, -14)$, and then continue increasing.

8. Sketch a graph of a function f that is differentiable and that satisfies the following conditions:
   - $f'(x) > 0$, when $x < -5$
   - $f'(x) < 0$, when $-5 < x < 1$ and when $x > 1$
   - $f'(-5) = 0$ and $f'(1) = 0$
   - $f(-5) = 6$ and $f(1) = 2$

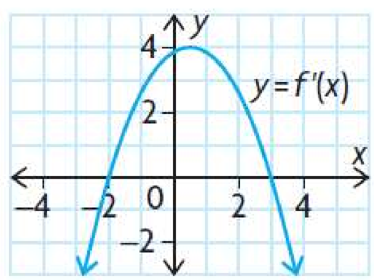
9. Each of the following graphs represents the derivative function $f'(x)$ of a function $f(x)$. Determine
   i. the intervals where $f(x)$ is increasing
   ii. the intervals where $f(x)$ is decreasing
   iii. the x-coordinate for all local extrema of $f(x)$

   Assuming that $f(0) = 2$, make a rough sketch of the graph of each function.

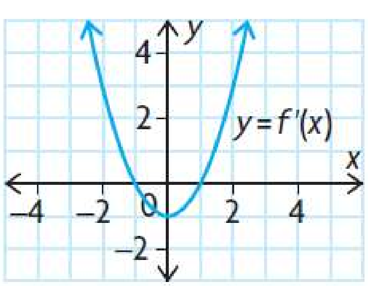
a.



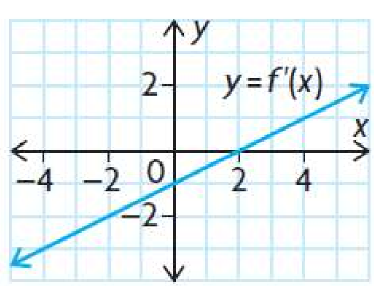
c.



b.



d.



10. Use the derivative to show that the graph of the quadratic function $f(x) = ax^2 + bx + c$, $a > 0$, is decreasing on the interval $x < -\frac{b}{2a}$ and increasing on the interval $x > -\frac{b}{2a}$.

11. For $f(x) = x^4 - 32x + 4$, find where $f'(x) = 0$, the intervals on which the function increases and decreases, and all the local extrema. Use graphing technology to verify your results.

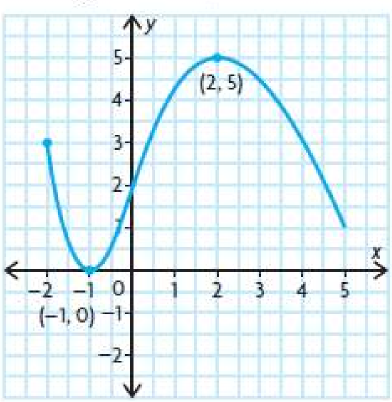
12. Sketch a graph of the function g that is differentiable on the interval $-2 \leq x \leq 5$, decreases on $0 < x < 3$, and increases elsewhere on the domain. The absolute maximum of g is 7, and the absolute minimum is -3. The graph of g has local extrema at $(0, 4)$ and $(3, -1)$.

13. Let f and g be continuous and differentiable functions on the interval $a \leq x \leq b$. If f and g are both increasing on $a \leq x \leq b$, and if $f(x) > 0$ and $g(x) > 0$ on $a \leq x \leq b$, show that the product fg is also increasing on $a \leq x \leq b$.

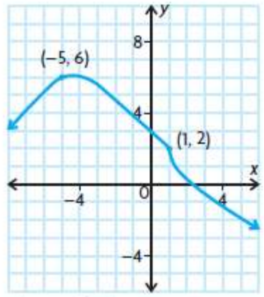
14. Let f and g be continuous and differentiable functions on the interval $a \leq x \leq b$. If f and g are both increasing on $a \leq x \leq b$, and if $f(x) < 0$ and $g(x) < 0$ on $a \leq x \leq b$, is the product fg increasing on $a \leq x \leq b$, decreasing, or neither?

Answers

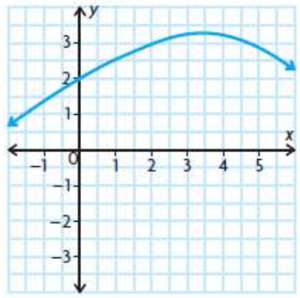
1. a. $(0, 1), (-4, 33)$
   b. $(0, 2)$
   c. $\left(\frac{1}{2}, 0\right), (2.25, -48.2), (-2, -125)$
   d. $\left(1, \frac{5}{2}\right), \left(-1, -\frac{5}{2}\right)$
2. A function is increasing when $f'(x) > 0$ and is decreasing when $f'(x) < 0$.
3. a. i. $x < -1, x > 2$
      ii. $-1 < x < 2$
      iii. $(-1, 4), (2, -1)$
   b. i. $-1 < x < 1$
      ii. $x < -1, x > 1$
      iii. $(-1, 2), (2, 4)$
   c. i. $x < -2$
      ii. $-2 < x < 2, 2 < x$
      iii. none
   d. i. $-1 < x < 2, 3 < x$
      ii. $x < -1, 2 < x < 3$
      iii. $(2, 3)$
4. a. increasing: $x < -2, x > 0$; decreasing: $-2 < x < 0$
   b. increasing: $x < 0, x > 4$; decreasing: $0 < x < 4$
   c. increasing: $x < -1, x > 1$; decreasing: $-1 < x < 0$, $0 < x < 1$
   d. increasing: $-1 < x < 3$; decreasing: $x < -1, x > 3$
   e. increasing: $-2 < x < 0, x > 1$; decreasing: $x < -2, 0 < x < 1$
   f. increasing: $x > 0$; decreasing: $x < 0$
5. increasing: $-3 < x < -2, x > 1$; decreasing: $x < -3, -2 < x < 1$
6.



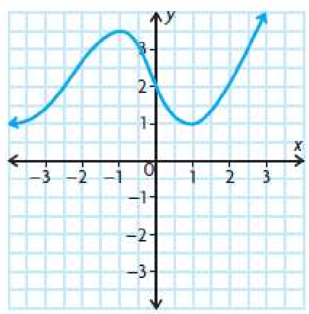
7. $a = 3, b = -9, c = -9$
8.



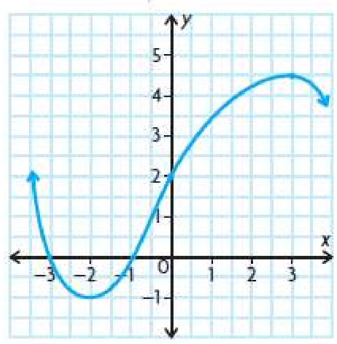
9. a. i. $x < 4$
      ii. $x > 4$
      iii. $x = 4$



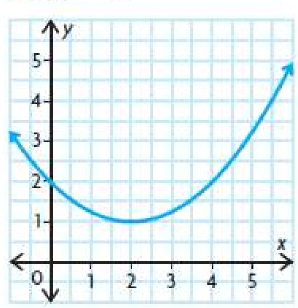
   b. i. $x < -1, x > 1$
      ii. $-1 < x < 1$
      iii. $x = -1, x = 1$



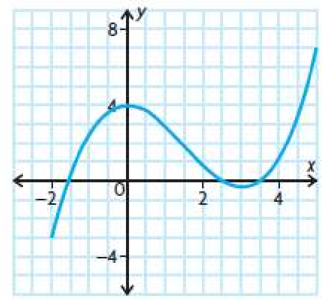
   c. i. $-2 < x < 3$
      ii. $x < -2, x > 3$
      iii. $x = -2, x = 3$



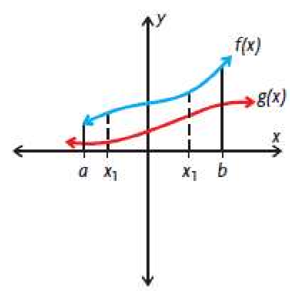
   d. i. $x > 2$
      ii. $x < 2$
      iii. $x = 2$



10. $f(x) = ax^2 + bx + c$
    $f'(x) = 2ax + b$
    Let $f'(x) = 0$, then $x = \frac{-b}{2a}$.
    If $x < \frac{-b}{2a}, f'(x) < 0$, therefore the function is decreasing.
    If $x > \frac{-b}{2a}, f'(x) > 0$, therefore the function is increasing.
11. $f'(x) = 0$ for $x = 2$,
    increasing: $x > 2$,
    decreasing: $x < 2$,
    local minimum: $(2, -44)$
12.



13. Let $y = f(x)$ and $u = g(x)$.
    Let x_1 and x_2 be any two values in the interval $a \le x \le b$ so that $x_1 < x_2$.
    Since $x_1 < x_2$, both functions are increasing:
    $f(x_2) > f(x_1)$ (1)
    $g(x_2) > g(x_1)$ (2)
    $yu = f(x) \cdot g(x)$
    (1) × (2) results in
    $f(x_2) \cdot g(x_2) > f(x_1)g(x_1)$
    The function yu or $f(x) \cdot g(x)$ is strictly increasing.



14. strictly decreasing