SOLUTIONS

MHF4U1 - UNIT 5 – RATIONAL FUNCTIONS, EQUATIONS AND INEQUALITIES

Supplementary Review Problems

1) Consider the following functions:

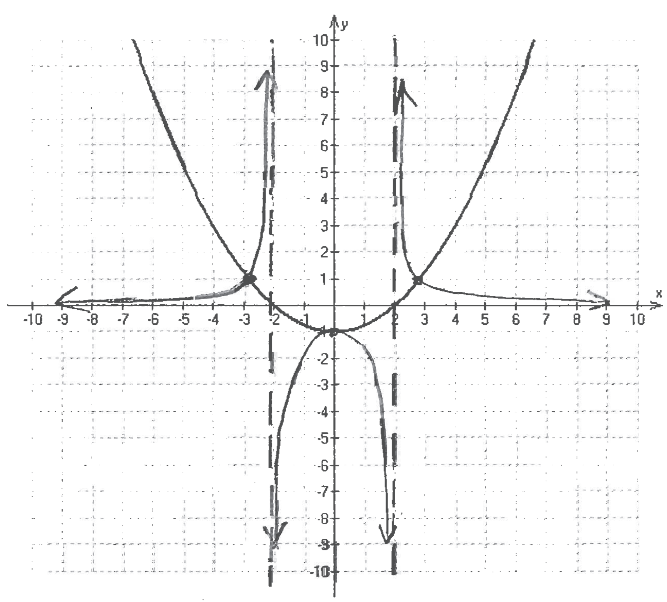
$f(x)=\dfrac{x^2+4x-21}{(x-3)(x-4)}=\dfrac{(x+7)(x-3)}{(x-3)(x-4)}$ $\quad g(x)=\dfrac{x-4}{(x-3)(x-4)}$ $\quad h(x)=\dfrac{7x^4+1}{3x^2+1}$

$p(x)=\dfrac{9x^3+8}{(x-3)(x-4)}$ $\quad q(x)=\dfrac{x+1}{(x-3)(x-4)}$

a) State the functions, if any, that do **not** have a vertical asymptote.
b) State the functions, if any, that have a horizontal asymptote.
c) State the functions, if any, that have a linear oblique asymptote.
d) State the functions, if any, that do **not** have an x-intercept.
e) State the functions, if any, that have a hole at $x = 3$.
f) State the functions, if any, that have a vertical asymptote at $x = 4$.

2) Solve the following equation: $\dfrac{x+3}{x-1}=2x+1$

3) The graph of a function $y = f(x)$ is shown below. On the graph given, sketch $y=\dfrac{1}{f(x)}$.



4) Determine the domain and range of the function $f(x) = \dfrac{1}{x^2 - 6x + 20}$.

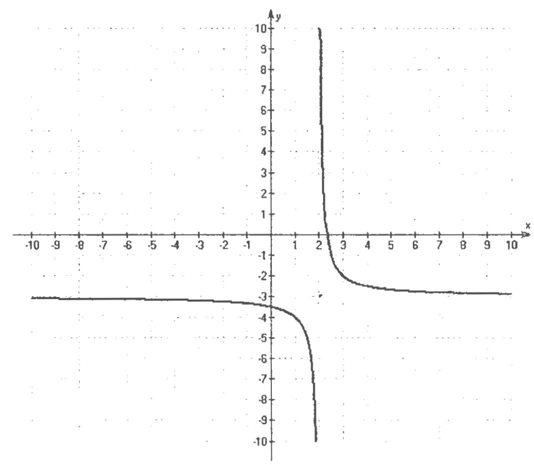
5) Determine the points, if any, at which the function $f(x) = x^2 - 8$ and its reciprocal intersect.

6) Sketch the graph of the function $f(x) = \dfrac{x^2 - 3x - 10}{x - 1}$. Be sure to show all work leading to your sketch and clearly show all intercepts and asymptotes.

7) Pedro bought a box of rare Math Wars comic books over the internet. He paid a total of \$750. Pedro kept 10 books for himself and sold the rest for a total of \$900, making \$20 profit for each book. How many books did Pedro initially buy?

8) Together, Joe and Jim can mow the lawn at a golf course in 6 hours. Working alone, Joe can mow the same lawn in 10 hours. How long would it take Jim to mow the lawn alone?

9) Azra graphed the reciprocal of a function *f*(*x*) and obtained the graph shown on the right. Determine the equation of *f*(*x*) (the original function) in simplified form.



10) Solve for *x* in the following inequality:

$$\frac{2x^2 + 5x - 3}{x^2 + x - 20} > 0.$$

11) Determine the intervals of increase and decrease for the function $f(x) = \dfrac{2x - 3}{x - 2}$.

12) Solve $\left|\dfrac{x - 2}{x}\right| \geq 3$.

13) The function $f(x) = \dfrac{x^3 + 2x^2 - 5x + 1}{x^2 + 3x + 5}$ has a linear oblique asymptote.

a) Determine the equation of the linear oblique asymptote.

b) Determine how (from above or below) the graph approaches the oblique asymptote as *x* approaches infinity and as *x* approaches negative infinity.

14) Find the equation of a function with vertical asymptotes $x = 2$ and $x = -3$, an *x*-intercept of 4, and a *y*-intercept of 4.

SOLUTIONS

#1) a) $h(x)$
b) $F(x)$, $g(x)$, $q(x)$
c) $p(x)$
d) $g(x)$, $h(x)$
e) $F(x)$
f) $F(x)$, $p(x)$, $q(x)$

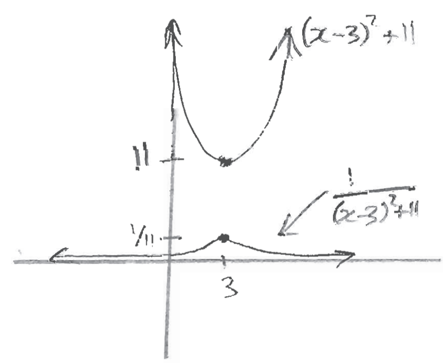
#2)
$$\frac{x+3}{x-1} = 2x+1 \quad , x \neq 1$$
$$x+3 = (2x+1)(x-1)$$
$$x+3 = 2x^2 - 2x + x - 1$$
$$0 = 2x^2 - 2x - 4$$
$$0 = 2(x^2 - x - 2)$$
$$0 = 2(x-2)(x+1)$$

$\therefore x = 2$ or $x = -1$

#3) See question page.

#4)
$$x^2 - 6x + 20$$
$$= x^2 - 6x + 9 - 9 + 20$$
$$= (x-3)^2 + 11$$

$$\therefore \frac{1}{x^2-6x+20} = \frac{1}{(x-3)^2+11}$$



From the graph, we see,

D: $\{x \in \mathbb{R}\}$
R: $\{y \in \mathbb{R} \mid 0 < y \leq \frac{1}{11}\}$

#5) $f(x)$ and its reciprocal intersect where $f(x) = 1$ or $f(x) = -1$

$x^2 - 8 = 1$	$x^2 - 8 = -1$
$x^2 = 9$	$x^2 = 7$
$x = \pm 3$	$x = \pm\sqrt{7}$

$\therefore$ intersection points are $(3,1)$, $(-3,1)$, $(\sqrt{7},-1)$, $(-\sqrt{7},-1)$

#6) $f(x) = \dfrac{x^2 - 3x - 10}{x - 1} = \dfrac{(x-5)(x+2)}{(x-1)}$

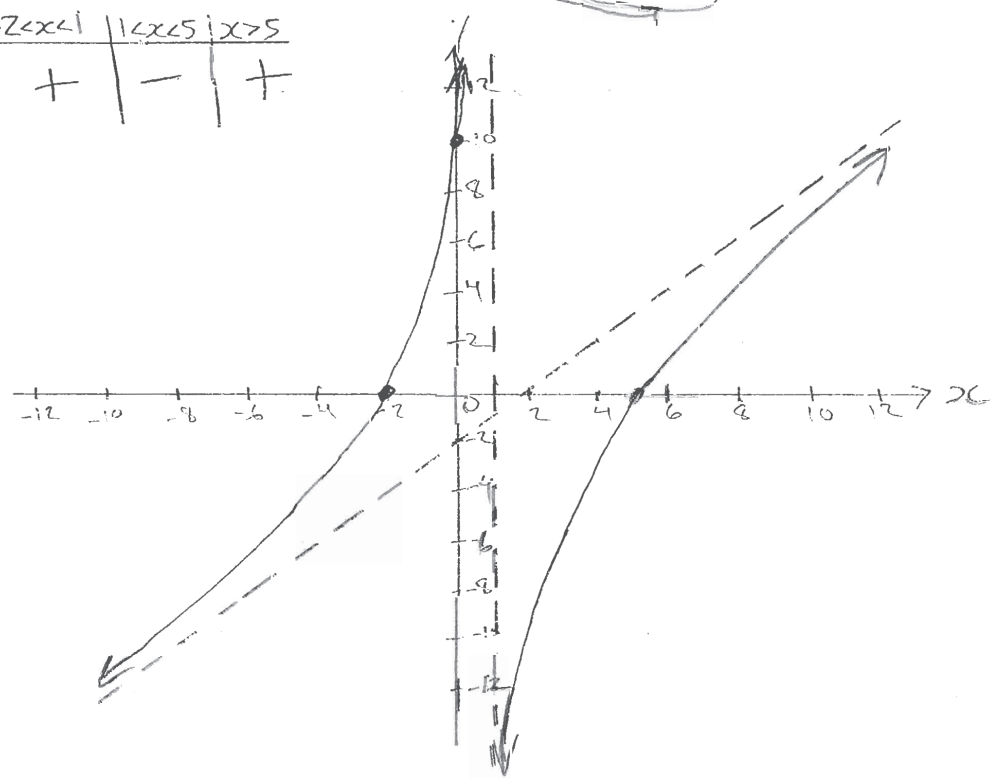
DOMAIN: $\{x \in \mathbb{R} \mid x \neq 1\}$

INTERCEPTS: x-int: 5 and -2 $\quad$ y-int: 10

ASYMPTOTES: Vertical: $x = 1$
Horizontal: none
Oblique: $y = x - 2$

$$\begin{array}{r|rrr} 1 & 1 & -3 & -10 \\ & & 1 & -2 \\ \hline & 1 & -2 & -12 \end{array}$$

Interval	$x<-2$	$-2<x<1$	$1<x<5$	$x>5$
Sign of $\dfrac{(x-5)(x+2)}{(x-1)}$	$-$	$+$	$-$	$+$



#7) Let x represent the initial # of books.

$$\frac{900}{x-10} - \frac{750}{x} = 20$$

$$x(x-10)\left(\frac{900}{x-10}\right) - x(x-10)\left(\frac{750}{x}\right) = 20x(x-10)$$

$$900x - 750x + 7500 = 20x^2 - 200x$$

$$0 = 20x^2 - 350x - 7500$$

$$0 = 10(2x^2 - 35x - 750)$$

$x = \cancel{-12.5}$ or $x = 30$ (using quadratic formula)

$\therefore$ Pedro initially bought 30 books.

#8) Let x represent the time it takes Jim to mow the lawn alone (in hours).

$$\frac{1}{x} + \frac{1}{10} = \frac{1}{6}$$

$$30\cancel{x}\left(\frac{1}{\cancel{x}}\right) + \overset{3}{\cancel{30}}x\left(\frac{1}{\cancel{10}}\right) = \overset{5}{\cancel{30}}x\left(\frac{1}{\cancel{6}}\right)$$

$$30 + 3x = 5x$$

$$30 = 2x$$

$$x = 15$$

$\therefore$ it would take Jim 15 hours to mow the lawn alone.

#9) Equation of given graph is $y = \frac{1}{x-2} - 3$

$\therefore$ equation of $f(x)$ is reciprocal of $y = \frac{1}{x-2} - 3$

Now, $\frac{1}{x-2} - 3$

$$= \frac{1}{x-2} - \frac{3(x-2)}{(x-2)}$$

$$= \frac{1-3(x-2)}{x-2}$$

$$= \frac{-3x+7}{x-2}$$

$$= \frac{7-3x}{x-2}$$

$$\therefore f(x) = \frac{x-2}{7-3x}$$

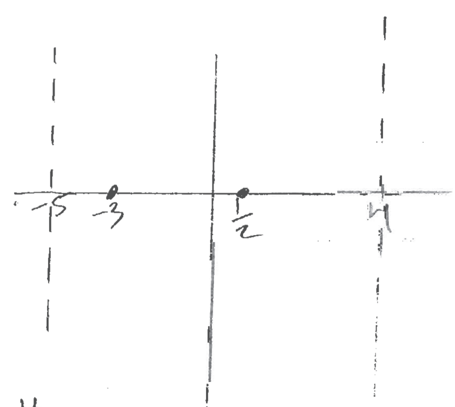
#10) $\frac{2x^2+5x-3}{x^2+x-20} > 0$

$\frac{(2x-1)(x+3)}{(x+5)(x-4)} > 0$

Domain: $\{x \in \mathbb{R} \mid x \neq -5, 4\}$

x-intercepts: $\frac{1}{2}, -3$

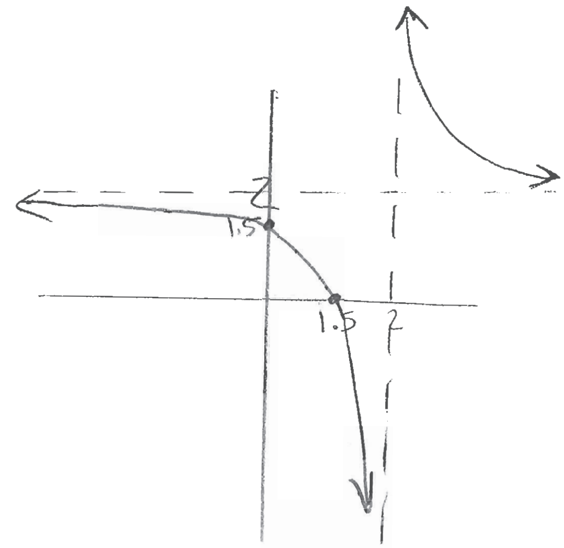
Vertical asymptotes: $x=-5$, $x=4$



Interval	$x<-5$	$-5<x<-3$	$-3<x<\frac{1}{2}$	$\frac{1}{2}<x<4$	$x>4$
Sign of $\frac{(2x-1)(x+3)}{(x+5)(x-4)}$	+	−	+	−	+

$\therefore \frac{2x^2+5x-3}{x^2+x-20} > 0$ when $x<-5$, $-3<x<\frac{1}{2}$, or $x>4$.

#11) $f(x) = \frac{2x-3}{x-2}$



Domain: $\{x \in \mathbb{R} \mid x \neq 2\}$
Intercepts: x-int: 1.5, y-int: 1.5
Asymptotes: Vertical: $x=2$
Horizontal: $y=2$
Oblique: None

Interval	$x<1.5$	$1.5<x<2$	$x>2$
Sign of $\frac{2x-3}{x-2}$	+	−	+

From the graph we see that the function is decreasing on $(-\infty, 2)$ and $(2, \infty)$. It is never increasing.

#12) $\left|\frac{x-2}{x}\right| \geq 3$ implies (1) $\frac{x-2}{x} \geq 3$ or (2) $\frac{x-2}{x} \leq -3$

(1) $\frac{x-2}{x} \geq 3$

$\frac{x-2}{x} - 3 \geq 0$

$\frac{x-2-3x}{x} \geq 0$

$\frac{-2x-2}{x} \geq 0$

$\frac{-2(x+1)}{x} \geq 0$

Interval	$x<-1$	$-1<x<0$	$x>0$
Sign of $\frac{-2(x+1)}{x}$	−	+	−

$\therefore \frac{x-2}{x} \geq 3$ when $-1 \leq x < 0$

CONTINUED

(2) $\frac{x-2}{x} \leq -3$

$\frac{x-2}{x} + 3 \leq 0$

$\frac{x-2+3x}{x} \leq 0$

$\frac{4x-2}{x} \leq 0$

$\frac{2(2x-1)}{x} \leq 0$

Interval	$x<0$	$0<x<\frac{1}{2}$	$x>\frac{1}{2}$
sign of $\frac{2(2x-1)}{x}$	+	−	+

$\therefore \frac{x-2}{x} \leq -3$ when $0<x\leq\frac{1}{2}$

$\therefore \left|\frac{x-2}{x}\right| \geq 3$ when $-1\leq x<0$ or $0<x\leq\frac{1}{2}$

#13) a)

$$\begin{array}{r|l} & x-1 \\ \hline x^2+3x+5 & x^3+2x^2-5x+1 \\ & \underline{x^3+3x^2+5x} \\ & -x^2-10x+1 \\ & \underline{-x^2-3x-5} \\ & -7x+6 \end{array}$$

$\therefore$ oblique asymptote is $y=x-1$

b) $f(x) = \frac{x^3+2x^2-5x+1}{x^2+3x+5} = x-1+\frac{-7x+6}{x^2+3x+5}$

For large positive x values, $\frac{-7x+6}{x^2+3x+5}$ is negative, so $x-1+\frac{-7x+6}{x^2+3x+5}$ is less than $x-1$. Therefore, as x approaches infinity, $f(x)$ approaches its oblique asymptote from below. For large negative x values, $\frac{-7x+6}{x^2+3x+5}$ is positive, so $x-1+\frac{-7x+5}{x^2+3x+5}$ is higher than $x-1$. Therefore, as x approaches negative infinity, $f(x)$ approaches its oblique asymptote from above.

#14) $y = \dfrac{a(x-4)}{(x-2)(x+3)}$

When $x=0$, $y=4$.

$\therefore \quad 4 = \dfrac{a(0-4)}{(0-2)(0+3)}$

$4 = \dfrac{-4a}{-6}$

$-24 = -4a$

$a = 6$

$\therefore$ equation is $y = \dfrac{6(x-4)}{(x-2)(x+3)}$