NAME: SOLUTIONS

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KNOW /10	APP /10	INQ /10	COMM /5

MCV4U1 - UNIT 8 – REPRESENTING LINES AND PLANES
TEST

1) For the line that passes through the points A(5, 8, -2) and B(-1, 3, 5), find

a) a vector equation. (*K – 2 marks*)

$\vec{d} = (-6, -5, 7)$ $\therefore (x, y, z) = (5, 8, -2) + t(-6, -5, 7)$

b) parametric equations. (*K –1 mark*)

$x = 5 - 6t$
$y = 8 - 5t$
$z = -2 + 7t$

2) Determine if and how the following lines intersect. If they intersect at a single point, find the point of intersection. (*K - 4 marks*)

L_1: $\frac{x-1}{4} = \frac{2-y}{5} = \frac{3-z}{1}$ $\quad \frac{y-2}{-5}$ $\quad \frac{z-3}{-1}$

L_2: $\frac{x-4}{1} = \frac{y}{-3} = \frac{3-z}{1}$ $\quad \frac{4-x}{-1}$ $\quad \frac{z-3}{-1}$

Not parallel

L_1: $x = 1 + 4t$
$y = 2 - 5t$
$z = 3 - t$

L_2: $x = 4 + s$
$y = -3s$
$z = 3 - s$

Where lines intersect,

$1 + 4t = 4 + s$ (1)
$2 - 5t = -3s$ (2)
$3 - t = 3 - s$ (3)

$4t - s = 3$ (1)
$-5t + 3s = -2$ (2)
$-t + s = 0$ (3)

From (3), $s = t$

Sub. in (1)
$4t - s = 3$
$4t - t = 3$
$3t = 3$
$t = 1$

$\therefore s = 1$

Check in (2)

L.S. $= -5t + 3s$
$= -5(1) + 3(1)$
$= -2$

R.S. $= -2$

$\therefore$ L.S. = R.S.

$\therefore$ lines intersect where $s = t = 1$
- point of intersection is $(5, -3, 2)$

3) Determine if and how the following line and plane intersect. If they intersect at a single point, determine the point of intersection. (*K – 3 marks*)

Line: $(x, y, z) = (4, -2, 3) + t(-1, 0, 9)$ Plane: $4x - 3y - z + 7 = 0$

$x = 4 - t$
$y = -2$
$z = 3 + 9t$

To find intersection:

$4(4-t) - 3(-2) - (3+9t) + 7 = 0$
$16 - 4t + 6 - 3 - 9t + 7 = 0$
$-13t = -26$
$t = 2$

∴ line and plane intersect where $t = 2$

$x = 4 - (2) = 2$
$y = -2$
$z = 3 + 9(2) = 21$

∴ line and plane intersect at the point $(2, -2, 21)$

4) The lines $(x, y, z) = (-3, 8, 1) + s(1, -1, 1)$ and $(x, y, z) = (1, 4, 2) + t(-3, k, 8)$ intersect at a point.

a) Determine the value of k. (*A – 2 marks*)

At point of intersection

① $-3 + s = 1 - 3t$
② $8 - s = 4 + kt$
③ $1 + s = 2 + 8t$

① $s + 3t = 4$
② $-s - kt = -4$
③ $s - 8t = 1$

① − ③: $11t = 3$, $t = \frac{3}{11}$

Sub. in ①: $s + 3\left(\frac{3}{11}\right) = 4$, $s = \frac{35}{11}$

Sub. in ②: $-\left(\frac{35}{11}\right) - k\left(\frac{3}{11}\right) = -4$
$-\frac{35}{11} + 4 = \frac{3k}{11}$
$k = 3$

b) Determine the coordinates of the point of intersection. (*A – 3 marks*)

From part (a), $t = \frac{3}{11}$.

From second equation:
$x = 1 - 3\left(\frac{3}{11}\right)$
$x = \frac{2}{11}$

$y = 4 + 3\left(\frac{3}{11}\right)$
$y = \frac{53}{11}$

$z = 2 + 8\left(\frac{3}{11}\right)$
$z = \frac{46}{11}$

∴ point of intersection is $\left(\frac{2}{11}, \frac{53}{11}, \frac{46}{11}\right)$

5) For each of the following sets of three planes, determine (by inspection) which diagram describes the intersection of the planes. Write the corresponding letter in the blank space provided. (*A – 5 marks*)

d

π_1: $x+2y+3z-4=0$
π_2: $3x+6y+9z-12=0$
π_3: $5x+10y+15z-20=0$

a

π_1: $x+2y+3z-4=0$
π_2: $3x+6y+9z-11=0$
π_3: $5x+10y+15z-16=0$

b

π_1: $x+2y+3z-4=0$
π_2: $3x+6y+9z-12=0$
π_3: $5x+10y+15z-16=0$

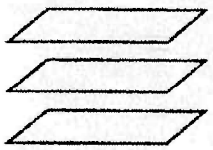
c

π_1: $x+2y+3z-4=0$
π_2: $3x+6y+9z-12=0$
π_3: $3x+5y-2z-1=0$

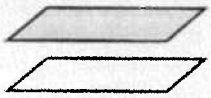
f

π_1: $x+2y+3z-4=0$
π_2: $3x+6y+9z-11=0$
π_3: $3x+5y-2z-1=0$

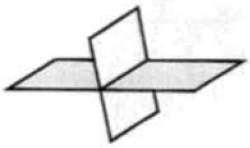
a)



b)

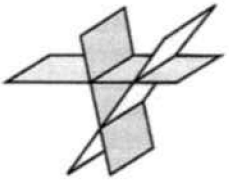


c)

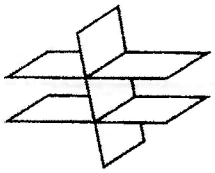


d)

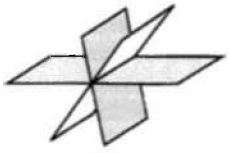
e)



f)



g)



6) A plane contains the points A(5, -2, 9), B(-3, 0, 6) and C(4, 1, -2).

a) Find a vector equation for the plane. (*1 – 3 marks*)

$\vec{d}_1 = (-8, 2, -3)$

$\vec{d}_2 = (-1, 3, -11)$

$\therefore (x, y, z) = (-3, 0, 6) + s(-8, 2, -3) + t(-1, 3, -11)$

b) Find a scalar equation for the plane. (*1 – 3 marks*)

$-8 \quad 2 \quad -3 \quad -8$
$-1 \quad 3 \quad -11 \quad -1$

$\vec{n} = \vec{d}_1 \times \vec{d}_2$

$= (-13, -85, -22)$

$\therefore -13x - 85y - 22z + D = 0$

$-13(-3) - 85(0) - 22(6) + D = 0$

$D = 93$

$\therefore -13x - 85y - 22z + 93 = 0$

or

$13x + 85y + 22z - 93 = 0$

7) Determine if and how the following planes intersect. If they intersect along a line, find the parametric equations of the line of intersection. (*1 – 4 marks*)

π_1: $3x - y + 4z - 7 = 0$ π_2: $x + y - 2z + 5 = 0$ Not parallel

$3x - y + 4z - 7 = 0$ ①

$x + y - 2z + 5 = 0$ ②

① + ② $4x + 2z - 2 = 0$

$2x + z - 1 = 0$

Let $x = t$

$\therefore 2(t) + z - 1 = 0$

$z = 1 - 2t$

→ Sub. in ②

$(t) + y - 2(1 - 2t) + 5 = 0$

$t + y - 2 + 4t + 5 = 0$

$y = -3 - 5t$

$\therefore$ planes intersect along the line

$x = t$

$y = -3 - 5t$

$z = 1 - 2t$

8) Determine if and how the following planes intersect. If they intersect at a single point, determine the point of intersection. If they intersect along a **single** line, find the parametric equations of the line of intersection. Otherwise, just state the nature of the intersection. (*C – 5 marks*)

π_1: $3x-3y-2z-14=0$ (1)
π_2: $5x+y-6z-10=0$ (2)
π_3: $x-2y+4z-9=0$ (3)

None are parallel

(1)+(2)×3 $\quad 18x-20z-44=0$
$9x-10z-22=0$ (4)

(2)×2+(3) $\quad 11x-8z-29=0$ (5)

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(4)×4−(5)×5 $\quad -19x+57=0$

$x=3$

Sub. in (4) $\quad 9(3)-10z-22=0$

$z=-\frac{1}{2}$

Sub in (3) $\quad (3)-2y+4\left(\frac{1}{2}\right)-9=0$

$3-2y+2-9=0$

$-2y=4$

$y=-2$

∴ planes intersect at the single point $\left(3,-2,-\frac{1}{2}\right)$