CARTESIAN (SCALAR) EQUATION OF A PLANE

Cartesian Equation of a Plane

At this point we have investigated vector and parametric equations of planes. We shall now develop an equation of a plane that does not involve parameters. This type of equation is called the **Cartesian equation** or **scalar equation** of a plane.

> **Normal Vector**
>
> A **normal vector** of a plane is a vector $\vec{n}$ that is perpendicular to the plane and, therefore, perpendicular to every vector in the plane.

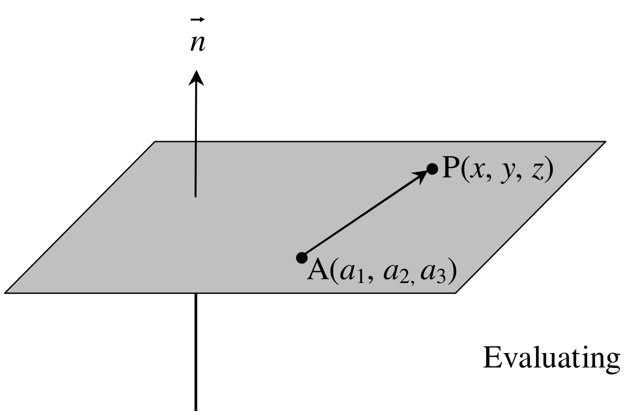
To develop the Cartesian equation of a plane, consider a plane with the following vector equation:

$$\vec{P} = \vec{A} + s\vec{d_1} + t\vec{d_2}$$

where

- $\vec{P} = (x, y, z)$ is the position vector for any point (x, y, z) on the plane
- $\vec{A} = (a_1, a_2, a_3)$ is the position vector for a fixed point (a_1, a_2, a_3) on the plane
- $\vec{d_1}$ and $\vec{d_2}$ are non-collinear direction vectors of the plane
- s and t are scalars

We can find a normal vector, $\vec{n} = (n_1, n_2, n_3)$, by calculating the cross product $\vec{d_1} \times \vec{d_2}$ (since $\vec{d_1} \times \vec{d_2}$ is perpendicular to $\vec{d_1}$ and $\vec{d_2}$, both of which lie on the plane).



$\vec{d_1} \times \vec{d_2} = \vec{n}$

$\overrightarrow{AP}$ is a vector in the plane. $\overrightarrow{AP} =$

Furthermore, $\overrightarrow{AP}$ and $\vec{n}$ are perpendicular.

$\therefore\ \vec{n} \bullet \overrightarrow{AP} =$

Evaluating $\vec{n} \bullet \overrightarrow{AP}$, we get

So, we have $n_1x + n_2y + n_3z \underbrace{- n_1a_1 - n_2a_2 - n_3a_3}_{\text{known number}} = 0$.

- $-n_1a_1 - n_2a_2 - n_3a_3$ is a known number…let's call it D.

$$\therefore\ n_1x + n_2y + n_3z + D = 0$$

The above equation is called the Cartesian equation or scalar equation of the plane.

- Notice that the coefficients of x, y and z are the components of the normal vector.

Cartesian (Scalar) Equation of a Plane

The **Cartesian equation** of a plane has the form $Ax + By + Cz + D = 0$, where A, B and C are the components of its normal vector, $\vec{n} = (A, B, C)$.

Example

Find a Cartesian equation of a plane that has normal vector (2, 6, -14) and that passes through the point (0, 3, 2).

Example
A plane has vector equation $(x, y, z) = (2, -5, 1) + s(2, -3, 0) + t(1, 1, -1)$. Find a scalar equation of the plane.

Example
A plane passes through the points A(2,7,1), B(1, 1, 1) and C(1, -3, 0).

a) Find a Cartesian equation of the plane

b) Determine if the point (3, 1, -2) lies on the plane.