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NAME: SOLUTIONS

KNOW /12	APP /12	INQ /12	COMM /6

MCV4U1 - UNIT 1 – INTRODUCTION TO CALCULUS
TEST

ROUND ALL ANSWERS TO THE NEAREST TENTH, UNLESS OTHERWISE STATED.

1) Evaluate each of the following limits. All answers should be exact. (*K – 1,2,2,3,2,2 marks*)

a) $\lim_{x\to 5}\frac{\sqrt{x+20}+25}{x+5}$

$= \frac{\sqrt{5+20}+25}{5+5}$

$= \frac{30}{10}$

$= 3$

b) $\lim_{x\to -4}\frac{3x^2+7x-20}{x^2+3x-4}$

$= \lim_{x\to -4}\frac{(3x-5)\cancel{(x+4)}}{\cancel{(x+4)}(x-1)}$

$= \frac{3(-4)-5}{-4-1}$

$= \frac{17}{5}$

c) $\lim_{x\to 10}\frac{2x-20}{\sqrt{x-6}-2} \times \frac{\sqrt{x-6}+2}{\sqrt{x-6}+2}$

$= \lim_{x\to 10}\frac{2(x-10)(\sqrt{x-6}+2)}{x-6-4}$

$= \lim_{x\to 10}\frac{2\cancel{(x-10)}(\sqrt{x-6}+2)}{\cancel{x-10}}$

$= 2(\sqrt{10-6}+2)$

$= 2(4)$

$= 8$

d) $\lim_{x\to 130}\frac{\sqrt[3]{x-5}-5}{x-130}$

Let $u = \sqrt[3]{x-5}$

$\therefore u^3 = x-5$

$u^3+5 = x$

As $x\to 130$, $u\to 5$

$= \lim_{u\to 5}\frac{u-5}{u^3+5-130}$

$= \lim_{u\to 5}\frac{u-5}{u^3-125}$

$= \lim_{u\to 5}\frac{\cancel{u-5}}{\cancel{(u-5)}(u^2+5u+25)}$

$= \frac{1}{5^2+5(5)+25}$

$= \frac{1}{75}$

e) $\lim_{x\to 8}\begin{cases}3x-5, & x<8\\ -x^2+2x+27, & x\geq 8\end{cases}$

f(x)

$\lim_{x\to 8^-} f(x) = 3(8)-5 = 19$

$\lim_{x\to 8^+} f(x) = -(8)^2+2(8)+27 = -21$

$\therefore \lim_{x\to 8} f(x)$ does not exist

f) $\lim_{x\to -2}\frac{4|x+2|}{x+2}$

$= \lim_{x\to -2}\begin{cases}\frac{-4(x+2)}{x+2}, & x<-2\\ \frac{4(x+2)}{x+2}, & x>-2\end{cases}$

f(x)

$= \lim_{x\to -2}\begin{cases}-4, & x<-2\\ 4, & x>-2\end{cases}$

$\lim_{x\to -2^-} f(x) = -4$, $\lim_{x\to -2^+} f(x) = 4$

$\therefore \lim_{x\to -2} f(x)$ does not exist

2) Use the difference quotient method to determine the equation of the tangent to the graph of $f(x) = x^2 - 2x + 13$ where $x = 3$. (*A – 4 marks*) $f(3) = 16$

$$m_{tan} = \lim_{h\to 0} \frac{f(3+h) - f(3)}{h}$$

$$= \lim_{h\to 0} \frac{(3+h)^2 - 2(3+h) + 13 - 16}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{9} + 6h + h^2 \cancel{-6} - 2h \cancel{-3}}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(h+4)}{\cancel{h}}$$

$$= 0 + 4$$

$$= 4$$

$\therefore$ $y = 4x + b$

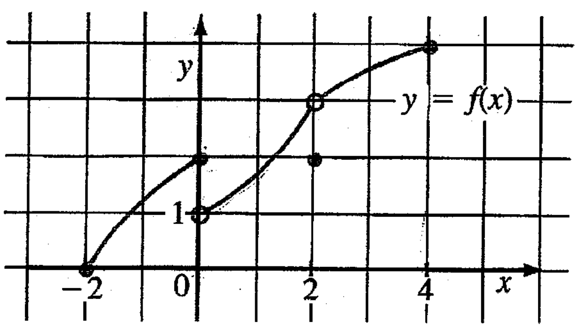
$16 = 4(3) + b$

$b = 4$

$\therefore$ equation is

$y = 4x + 4$

3) For the graph of $y = f(x)$ shown below, determine each of the following limits. (*A – 4 marks*)



a) $\lim_{x\to 2^-} f(x) =$ 3

b) $\lim_{x\to 2^+} f(x) =$ 3

c) $\lim_{x\to 2} f(x) =$ 3

d) $\lim_{x\to 0} f(x) =$ does not exist

4) As a snowball melts, it's volume decreases. The volume in cubic centimetres is given by the equation $V(r) = \frac{4}{3}\pi r^3$, where r is the radius in centimetres. Determine the rate of change of the snowball's volume when its radius is 10 cm. (*A – 4 marks*)

$$I.R.O.C. = \lim_{h\to 0} \frac{V(10+h) - V(10)}{h}$$

$$= \lim_{h\to 0} \frac{\frac{4}{3}\pi(10+h)^3 - \frac{4}{3}\pi(10)^3}{h}$$

$$= \frac{4}{3}\pi \lim_{h\to 0} \frac{h^3 + 30h^2 + 300h + 1000 - 1000}{h}$$

$$= \frac{4}{3}\pi \lim_{h\to 0} \frac{\cancel{h}(h^2 + 30h + 300)}{\cancel{h}}$$

$$= \frac{4}{3}\pi\left[0^2 + 30(0) + 300\right]$$

$$= \frac{4}{3}\pi(300)$$

$$= 400\pi \doteq 1256.6 \text{ cm}^3/\text{cm}$$

5) For the following function, $\lim_{x \to -2} f(x) = 8$. Determine the exact values of *a* and *b*. (*I – 6 marks*)

$$f(x) = \begin{cases} -3ax^3 - 4b, & x \le -2 \\ ax + b, & x > -2 \end{cases}$$

$\lim_{x \to -2^-} f(x) = 8$

$-3a(-2)^3 - 4b = 8$

$24a - 4b = 8$

① $6a - b = 2$

$\lim_{x \to -2^+} f(x) = 8$

$a(-2) + b = 8$

② $-2a + b = 8$

$\therefore$ ① $6a - b = 2$

② $-2a + b = 8$

①+② $4a = 10$

$a = 2.5$

sub. in ②

$-2(2.5) + b = 8$

$b = 8 + 5$

$b = 13$

$\therefore a = 2.5$

$b = 13$

6) Determine the **coordinates** of the points on the graph of $f(x) = x^3 + 3x^2 - 21x + 2$ at which the tangent is parallel to the line $6x - 2y = 19$. (*I – 6 marks*)

$6x - 2y = 19$

$y = 3x - \frac{19}{2}$

$\therefore$ need slope of 3

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = 3$$

$$\lim_{h \to 0} \frac{(x+h)^3 + 3(x+h)^2 - 21(x+h) + 2 - (x^3 + 3x^2 - 21x + 2)}{h} = 3$$

$$\lim_{h \to 0} \frac{\cancel{x^3} + 3x^2h + 3xh^2 + h^3 + \cancel{3x^2} + 6xh + 3h^2 - \cancel{21x} - 21h + \cancel{2} - \cancel{x^3} - \cancel{3x^2} + \cancel{21x} - \cancel{2}}{h} = 3$$

$$\lim_{h \to 0} \frac{\cancel{h}(3x^2 + 3xh + h^2 + 6x + 3h - 21)}{\cancel{h}} = 3$$

$3x^2 + 6x - 21 = 3$

$3x^2 + 6x - 24 = 0$

$x^2 + 2x - 8 = 0$

$(x+4)(x-2) = 0$

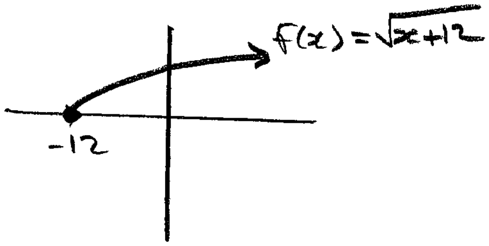
$x = -4$ or $x = 2$

$\therefore$ points are

$(-4, 70)$ and $(2, -20)$

7) While engaged in an intense conversation about limits, Karlo claimed that $\lim_{x \to -12} \sqrt{x+12}$ is equal to 0. Is Karlo's claim correct? Explain. (*C – 3 marks*)

Karlo's claim is incorrect. The domain of $f(x)=\sqrt{x+12}$ is $\{x \in \mathbb{R} \mid x \geq -12\}$. Therefore, $\lim_{x \to -12^-} \sqrt{x+12}$ does not exist, whereas $\lim_{x \to -12^+} \sqrt{x+12} = 0$. So, $\lim_{x \to -12} \sqrt{x+12}$ does not exist.



8) A few days after their intense conversation about limits, Jenny and Karlo went on a date. While they were enjoying their meal, Jenny told Karlo that the expression $\frac{f(x)-f(5)}{x-5}$ is the slope of the tangent line to $f(x)$ at the point where $x = 5$. Is Jenny's claim correct? Explain. (*C – 3 marks*)

Jenny's claim is incorrect. $\frac{f(x)-f(5)}{x-5}$ gives the slope of a secant line. The slope of the tangent line is given by $\lim_{x \to 5} \frac{f(x)-f(5)}{x-5}$.

limit is needed