Polynomial Division…A Deeper Look

Before we investigate the workings of polynomial division, we need to consider another way of writing numbers.

We can express real numbers as a sum of multiples of powers of ten.

Examples

Express each of the following as a sum of multiples of powers of ten.

a) 524 =

b) 9108.25 =

To warm up to the ideas behind long division of polynomials, let's try some division using numbers written in the form described above.

First, let's consider a simple example of $824 \div 2$.

Using Powers of Ten

A Similar Algebraic Example

The above example begins to show how division with polynomials relates to division of real numbers.

Making the Jump to Long Division

Long division is ultimately a series of approximations. That is, we break our dividend into parts and deal with each part separately. Consider the following example:

$2\overline{)758}$

We start by asking "how many times does 2 go into 7?" The real question here, however, is "how many times does 2 go into 700?" Notice that we've approximated our dividend to be 700.

$$\begin{array}{r} 3 \\ 2\overline{)758} \end{array}$$

The answer is 3, which actually represents 300. We've just made an approximation! We know that 2 goes into 700 at least 300 times. Note that an answer of 4 (representing 400) would be too high.

$$\begin{array}{l} \;\;\;3 \\ 2\overline{)758} \\ \;\;\;\underline{6} \\ \;\;\;1 \end{array}$$

The next step is to reduce our approximated dividend by 600, since we know that at least 300 groups of 2 fit into 700. Notice that the 6 actually represents 600 and the 1 represents 100.

$$\begin{array}{l} \;\;\;3 \\ 2\overline{)758} \\ \;\;\;\underline{6} \\ \;\;\;15 \end{array}$$

Now, we our next task is to determine how many times 2 goes into the remaining 158. Again, we approximate by asking how many times 2 goes into 150, which is represented in our long division by the 15. “Bringing down the 5” simply refines our approximation of the remaining dividend from 100 to 150.

$$\begin{array}{l} \;\;\;37 \\ 2\overline{)758} \\ \;\;\;\underline{6} \\ \;\;\;15 \end{array}$$

Even though we may be thinking “2 goes into 15 seven times,” we are actually stating that 2 goes into 150 approximately 70 times.

$$\begin{array}{l} \;\;\;37 \\ 2\overline{)758} \\ \;\;\;\underline{6} \\ \;\;\;15 \\ \;\;\;\underline{14} \\ \;\;\;\;\;1 \end{array}$$

Once again, we reduce our approximated dividend, this time by 140 (shown as 14), since we know that at least 70 groups of 2 fit into 150.

$$\begin{array}{l} \;\;\;37 \\ 2\overline{)758} \\ \;\;\;\underline{6} \\ \;\;\;15 \\ \;\;\;\underline{14} \\ \;\;\;\;\;18 \end{array}$$

Finally, we refine our remaining approximated dividend of 10 (shown as 1) by “bringing down the 8”.

$$\begin{array}{l} \;\;\;379 \\ 2\overline{)758} \\ \;\;\;\underline{6} \\ \;\;\;15 \\ \;\;\;\underline{14} \\ \;\;\;\;\;18 \\ \;\;\;\;\;\underline{18} \\ \;\;\;\;\;\;\;0 \end{array}$$

At this point we are dividing into our remaining dividend of 18. 2 divides into 18 exactly 9 times, thus we will have no remainder (the final step has been shown to verify that the remainder is 0).

Powers of Ten and Polynomials

Now that we have taken a closer look at the inner workings of numerical long division, we can begin to analyze polynomial long division. We will bridge the gap between numerical long division and algebraic long division by expressing numerical division in terms of multiples of powers of 10.

Example

Standard Long Division	Using Powers of 10	Similar Algebraic Example
$12\overline{)384}$		

A More Interesting Example

Standard Long Division	Using Powers of 10	Similar Algebraic Example
$231\overline{)4265}$		

A Quick Look at Synthetic Division

To understand how the process of long division of polynomials is simplified using synthetic division, we can simply compare both methods for a given division.

Example

$(3x^2+5x-22)\div(x-2)$

Long Division	Synthetic Division

Example

$(4x^3-7x^2+5x-12)\div(x+5)$

Long Division	Synthetic Division