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NAME: Solutions

KNOW	/ 12	APP	/ 12	INQ	/ 12	COMM	/ 6

MHF4U1 - UNIT 7 – TRIGONOMETRIC IDENTITIES AND EQUATIONS
TEST

1) **Using trigonometric identities**, determine the exact value of each of the following. **SHOW ALL WORK.** (*K – 3 marks each*)

a) $\sin(22.5°)$

$\cos 2(22.5°) = 1 - 2\sin^2(22.5°)$

$\cos 45° = 1 - 2\sin^2(22.5°)$

$\frac{\sqrt{2}}{2} = 1 - 2\sin^2 22.5°$

$2\sin^2 22.5° = 1 - \frac{\sqrt{2}}{2}$

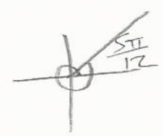
$2\sin^2 22.5° = \frac{2-\sqrt{2}}{2}$

$\sin^2 22.5° = \frac{2-\sqrt{2}}{4}$

$\sin 22.5° = \cancel{\pm}\frac{\sqrt{2-\sqrt{2}}}{2}$

(negative answer is inadmissible)

b) $\cos\left(-\frac{29\pi}{12}\right) = \cos\frac{29\pi}{12}$



$= \cos\frac{5\pi}{12}$

$= \cos\left(\frac{2\pi}{12} + \frac{3\pi}{12}\right)$

$= \cos\left(\frac{\pi}{6} + \frac{\pi}{4}\right)$

$= \cos\frac{\pi}{6}\cos\frac{\pi}{4} - \sin\frac{\pi}{6}\sin\frac{\pi}{4}$

$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

$= \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}$

$= \frac{\sqrt{6}-\sqrt{2}}{4}$

2) Simplify the following expression as much as possible.

$$\cos\left(\frac{\pi}{2} - x\right) + \sin\left(\frac{\pi}{2} + x\right) - \cos(\pi + x) + 4\cos\left(\frac{3\pi}{2} - x\right)$$

(*K – 3 marks*)

$= \sin x + \cos x - (-\cos x) + (-4\sin x)$

$= 2\cos x - 3\sin x$

3) Express the following as a **single trigonometric ratio** and **evaluate**. (*K – 3 marks*)

$$\cos\frac{3\pi}{8}\cos\frac{7\pi}{8} - \sin\frac{3\pi}{8}\sin\frac{7\pi}{8}$$

$= \cos\left(\frac{3\pi}{8} + \frac{7\pi}{8}\right)$

$= \cos\frac{10\pi}{8}$

$= \cos\frac{5\pi}{4}$

$= -\cos\frac{\pi}{4}$

$= -\frac{1}{\sqrt{2}}$

$= -\frac{\sqrt{2}}{2}$

4) Determine the solutions for each equation on the interval $0° \le x \le 360°$. Give exact solutions, where possible. Round approximate solutions to the nearest tenth of a degree. (*A - 3 marks each*)

a) $4\sin x + 3 = \sin x + 1$

$3\sin x = -2$

$\sin x = -\frac{2}{3}$

$R.A.A \doteq 41.8°$

Quad. III + IV

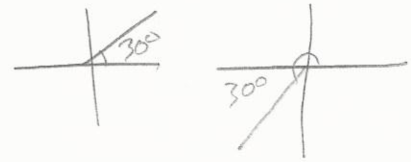
$x = 221.8°,\ 318.2°$

b) $\sqrt{3}\tan 2x - 1 = 0$

$\tan 2x = \frac{1}{\sqrt{3}}$ $\qquad 0° \le 2x \le 720°$

$RAA = 30°$

Quad I + III



$2x = 30°, 390°$ $\quad 2x = 210°, 570°$

$\therefore x = 15°, 105°, 195°, 285°$

5) Determine the solutions for each equation on the interval $0 \le x \le 2\pi$. Give exact solutions, where possible. Round approximate solutions to the nearest tenth of a radian. (*A - 3 marks each*)

a) $-6\sin^2 x = \cos x - 5$

$-6(1-\cos^2 x) = \cos x - 5$

$-6 + 6\cos^2 x = \cos x - 5$

$6\cos^2 x - \cos x - 1 = 0$

$(3\cos x + 1)(2\cos x - 1)$

$3\cos x + 1 = 0$ or $2\cos x - 1 = 0$

$\cos x = -\frac{1}{3}$ $\qquad \cos x = \frac{1}{2}$

$R.A.A \doteq 1.23$ $\qquad RAA = \frac{\pi}{3}$

Quad. II + III $\qquad$ Quad I + IV

$x \doteq 1.9,\ 4.4$ $\qquad x = \frac{\pi}{3},\ \frac{5\pi}{3}$

$\therefore x = \frac{\pi}{3}, 1.9, 4.4, \frac{5\pi}{3}$

b) $\sec x \tan x = 4.7\tan x$

$\sec x\tan x - 4.7\tan x = 0$

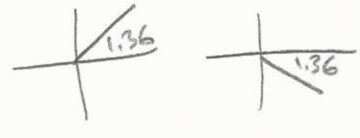
$\tan x(\sec x - 4.7) = 0$

$\tan x = 0$ or $\sec x = 4.7$

$x = 0, \pi, 2\pi$ $\qquad \cos x = \frac{1}{4.7}$

$R.A.A. \doteq 1.36$

Quad I + IV



$x = 1.4$ or 4.9

$\therefore x = 0, 1.4, \pi, 4.9, 2\pi$

6) Prove the following identities. (*I - 4 marks each*)

a) $1+\cot x \tan y = \dfrac{\sin(x+y)}{\sin x \cos y}$

L.S.

$1+\cot x \tan y$

$= 1+\left(\dfrac{\cos x}{\sin x}\right)\left(\dfrac{\sin y}{\cos y}\right)$

$= 1+\dfrac{\cos x \sin y}{\sin x \cos y}$

$= \dfrac{\sin x \cos y}{\sin x \cos y} + \dfrac{\cos x \sin y}{\sin x \cos y}$

$= \dfrac{\sin x \cos y + \cos x \sin y}{\sin x \cos y}$

R.S.

$\dfrac{\sin(x+y)}{\sin x \cos y}$

$= \dfrac{\sin x \cos y + \cos x \sin y}{\sin x \cos y}$

$\therefore$ L.S = R.S

$\therefore$ identity proven

b) $\dfrac{1+\sin 2x+\cos 2x}{1+\sin 2x-\cos 2x} = \dfrac{\tan x \cos^2 x}{1-\cos^2 x}$

L.S.

$\dfrac{1+\sin 2x+\cos 2x}{1+\sin 2x-\cos 2x}$

$= \dfrac{1+2\sin x \cos x + 2\cos^2 x - 1}{1+2\sin x \cos x - (1-2\sin^2 x)}$

$= \dfrac{2\sin x \cos x + 2\cos^2 x}{2\sin x \cos x + 2\sin^2 x}$

$= \dfrac{\cancel{2}\cos x\,\cancel{(\sin x+\cos x)}}{\cancel{2}\sin x\,\cancel{(\cos x+\sin x)}}$

$= \dfrac{\cos x}{\sin x}$

R.S.

$\dfrac{\tan x \cos^2 x}{1-\cos^2 x}$

$= \dfrac{\dfrac{\sin x}{\cancel{\cos x}}(\cos^{\cancel{2}} x)}{\sin^2 x}$

$= \dfrac{\cancel{\sin x}\cos x}{\sin^2 x}$

$= \dfrac{\cos x}{\sin x}$

$\therefore$ L.S. = R.S.

$\therefore$ identity proven

7) Determine the solutions to the following equation on the interval $-3\pi \le x \le 3\pi$. Round all final answers to the nearest tenth of a radian. *(I – 6 marks)*

$$5\sin x - 2\cos^2 x = 4$$

$5\sin x - 2(1-\sin^2 x) = 4$

$5\sin x - 2 + 2\sin^2 x = 4$

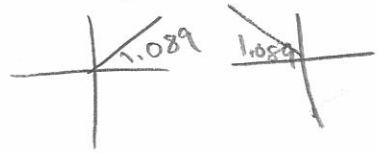
$2\sin^2 x + 5\sin x - 6 = 0$

$\sin x = \dfrac{-5 \pm \sqrt{5^2 - 4(2)(-6)}}{2(2)}$

$= \dfrac{-5 \pm \sqrt{73}}{4}$

$\sin x = 3.386$
No solution

$\sin x = 0.886$
R.A.A $= 1.089$
sin is positive.
Quadrant I or II



$x = 1.089, 7.372, -5.194$ $\quad x = 2.052, 8.336, -4.231$

$\therefore x \doteq -5.2, -4.2, 1.1, 2.1, 7.4, 8.3$

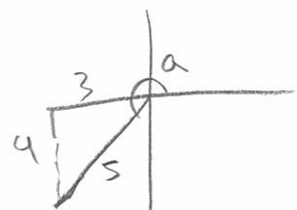
8) Let $\pi \le a \le \dfrac{3\pi}{2}$ and $\dfrac{3\pi}{2} \le b \le 2\pi$. If $\sin a = -\dfrac{4}{5}$ and $\cos b = \dfrac{12}{13}$, find the exact value of $\cos(a-b)$.
(C - 3 marks)

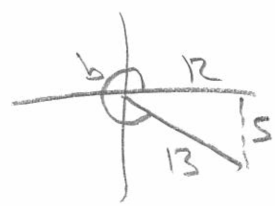
$\cos(a-b) = \cos a\cos b + \sin a \sin b$

$= \left(-\frac{3}{5}\right)\left(\frac{12}{13}\right) + \left(-\frac{4}{5}\right)\left(-\frac{5}{13}\right)$

$= \dfrac{-36}{65} + \dfrac{20}{65}$

$= -\dfrac{16}{65}$





9) Derive the formula $\cos 2x = 2\cos^2 x - 1$. **<u>Do not</u>** start your derivation with a cosine double angle formula. *(C - 3 marks)*

$\cos 2x = \cos(x+x)$

$= \cos x\cos x - \sin x \sin x$

$= \cos^2 x - \sin^2 x$

$= \cos^2 x - (1-\cos^2 x)$

$= \cos^2 x - 1 + \cos^2 x$

$= 2\cos^2 x - 1$