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NAME: SOLUTIONS

KNOW /12	APP /12	INQ /12	COMM /6

MCV4U1 - UNIT 7 – APPLICATIONS OF VECTORS

TEST

ROUND ALL FINAL ANSWERS TO THE NEAREST TENTH, UNLESS OTHERWISE STATED.

1) Consider the vectors $\vec{u} = (2, 3, -5)$, $\vec{v} = (-3, -2, 6)$ and $\vec{w} = (-4, 8, 4)$. *(K – 2 marks each)*

a) Calculate $\vec{u} \cdot \vec{v}$

$\vec{u} \cdot \vec{v} = -6 - 6 - 30$

$= -42$

b) Calculate $\vec{u} \times \vec{v}$

$\begin{matrix} 2 & 3 & -5 & 2 \\ -3 & -2 & 6 & -3 \end{matrix}$

$\vec{u} \times \vec{v} = (8, 3, 5)$

c) Calculate $\vec{u} \downarrow \vec{v}$

$\vec{u} \downarrow \vec{v} = \frac{\vec{u} \cdot \vec{v}}{\vec{v} \cdot \vec{v}} \vec{v}$

$= \frac{-42}{49}(-3, -2, 6)$

$= -\frac{6}{7}(-3, -2, 6)$

$= \left(\frac{18}{7}, \frac{12}{7}, -\frac{36}{7}\right)$

d) Determine if $\vec{u}$, $\vec{v}$ and $\vec{w}$ are coplanar.

$\vec{u} \cdot \vec{v} \times \vec{w}$

$= (2, 3, -5) \cdot (-3, -2, 6) \times (-4, 8, 4)$

$= (2, 3, -5) \cdot (-56, -12, -32)$

$= 12$

$\neq 0$

$\therefore$ not coplanar

$\begin{matrix} -3 & -2 & 6 & -3 \\ -4 & 8 & 4 & -4 \end{matrix}$

e) Determine the angle between $\vec{v}$ and $\vec{w}$.

$\vec{v} \cdot \vec{w} = |\vec{v}||\vec{w}|\cos\theta$

$20 = (7)(\sqrt{96})\cos\theta$

$\frac{20}{7\sqrt{96}} = \cos\theta$

$\underline{\underline{\theta = 73.0^\circ}}$

f) Determine the direction cosines of $\vec{w}$.

$\cos\alpha = -\frac{4}{\sqrt{96}}$

$= -\frac{4}{4\sqrt{6}}$

$= -\frac{\sqrt{6}}{6}$

$\cos\beta = \frac{8}{\sqrt{96}}$

$= \frac{8}{4\sqrt{6}}$

$= \frac{\sqrt{6}}{3}$

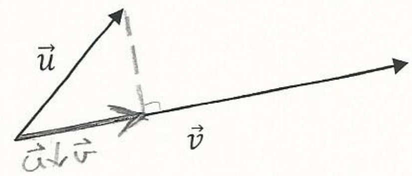
$\cos\gamma = \frac{4}{\sqrt{96}}$

$= \frac{4}{4\sqrt{6}}$

$= \frac{\sqrt{6}}{6}$

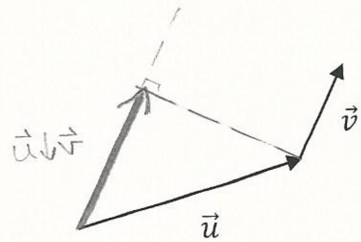
2) For each pair of vectors shown below,
   a) draw $\vec{u} \downarrow \vec{v}$. *(A – 2 marks)*
   b) state whether $\vec{u} \times \vec{v}$ acts **into the page** or **out of the page**. *(A – 2 marks)*

(i)



Into the page

(ii)

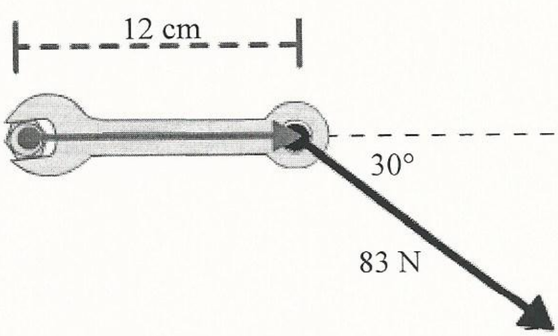


Out of the page

3) Mr. Gesjorskyj attaches a rope to his donkey. The rope makes an angle of 35° with the vertical. He proceeds to pull the donkey with a force of 855 N for a distance of 100 m, breaking the world record for the 100 m dash in the process, completing the job in 9.47 seconds. Calculate the work that Mr. Gesjorskyj did. *(A – 2 marks)*

$W = \vec{F} \cdot \vec{d}$
$= |\vec{F}||\vec{d}|\cos\theta$
$= (855)(100)\cos 55°$
$\doteq 49\,040.8 \text{ J}$

4) A force of 83 N is applied to a wrench as shown below. Determine the torque produced. *(A – 2 marks)*



$|\vec{T}| = |\vec{r} \times \vec{F}|$
$= |\vec{r}||\vec{F}|\sin\theta$
$= (0.12)(83)\sin 30°$
$= 4.98 \text{ N}\cdot\text{m}$ into the page

5) Given that $\vec{u} = (3m, 2m - 4, 1)$ and $\vec{v} = (m, 5, 3m + 10)$ are perpendicular, determine all possible values of m. *(A – 2 marks)*

$\vec{u} \cdot \vec{v} = 0$ since perpendicular
$3m^2 + 10m - 20 + 3m + 10 = 0$
$3m^2 + 13m - 10 = 0$
$(3m - 2)(m + 5) = 0$

$\therefore m = \frac{2}{3}$ or $m = -5$

6) State whether each expression is a vector, a scalar, or meaningless. *(A – 2 marks)*

a) $(\vec{a} \times \vec{b}) \times (\vec{u} \times \vec{v})$ vector

b) $|\vec{u}|(\vec{a} \cdot \vec{b})$ scalar

7) Given the vectors $\vec{u} = (2, 4, -5)$ and $\vec{v} = (-2, -3, 1)$, determine a **unit vector** that is perpendicular to both $\vec{u}$ and $\vec{v}$. *(I – 2 marks)*

$$\begin{array}{cccc} 2 & 4 & -5 & 2 \\ -2 & -3 & 1 & -2 \end{array}$$

$\vec{u} \times \vec{v} = (-11, 8, 2)$

$|\vec{u} \times \vec{v}| = \sqrt{189}$

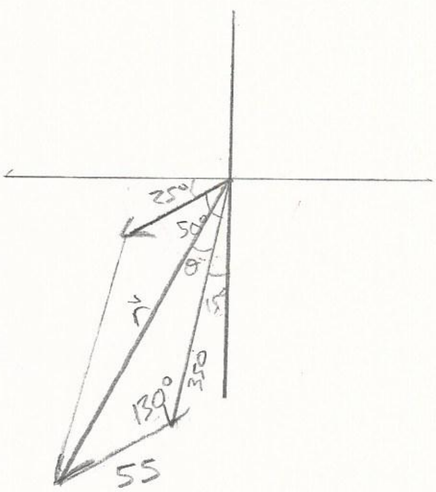
$= 3\sqrt{21}$

$\therefore$ unit vector is

$\left(\frac{-11}{3\sqrt{21}}, \frac{8}{3\sqrt{21}}, \frac{2}{3\sqrt{21}}\right)$

$= \left(\frac{-11\sqrt{21}}{63}, \frac{8\sqrt{21}}{63}, \frac{2\sqrt{21}}{63}\right)$

8) An airplane is flying at 350 km/h on a heading of 195°. A wind is blowing at a speed of 55 km/h in the direction S65°W. Find the resultant groundspeed of the airplane. *(I – 4 marks)*



$|\vec{r}|^2 = 55^2 + 350^2 - 2(55)(350)\cos 130°$

$|\vec{r}| \doteq 387.6$ km/h

$\frac{\sin\theta}{55} = \frac{\sin 130°}{387.6}$

$\theta \doteq 6.2°$

$180° + 15° + 6.2° = 201.2°$

$\therefore$ groundspeed is 387.6 km/h on a heading of 201.2°

9) If $\beta = 60°$ and $\gamma = 135°$ for a given vector, determine all possible values for α. *(I – 3 marks)*

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\cos^2\alpha + \cos^2 60° + \cos^2 135° = 1$

$\cos^2\alpha + \frac{1}{4} + \frac{1}{2} = 1$

$\cos^2\alpha = \frac{1}{4}$

$\cos\alpha = \pm\frac{1}{2}$

$\therefore \alpha = 60°$ or $120°$

10) If $|\vec{u}| = 3$, $|\vec{v}| = 2\sqrt{2}$ and the angle between $\vec{u}$ and $\vec{v}$ is 45°, find the value of $|2\vec{u} - 3\vec{v}|$. (Hint: $\vec{w} \cdot \vec{w} = |\vec{w}|^2$) *(I – 3 marks)*

$|2\vec{u}-3\vec{v}|^2 = (2\vec{u}-3\vec{v})\cdot(2\vec{u}-3\vec{v})$

$|2\vec{u}-3\vec{v}|^2 = 4|\vec{u}|^2 - 12\vec{u}\cdot\vec{v} + 9|\vec{v}|^2$

$|2\vec{u}-3\vec{v}|^2 = 4(3)^2 - 12|\vec{u}||\vec{v}|\cos\theta + 9(2\sqrt{2})^2$

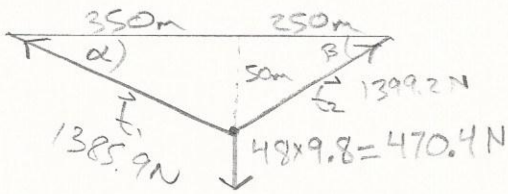
$|2\vec{u}-3\vec{v}|^2 = 36 - 12(3)(2\sqrt{2})\cos 45° + 9(8)$

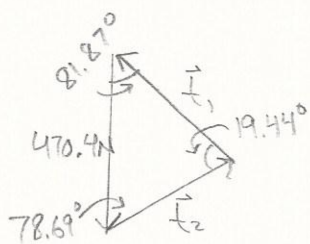
$|2\vec{u}-3\vec{v}|^2 = 36 - 72 + 72$

$|2\vec{u}-3\vec{v}|^2 = 36$

$|2\vec{u}-3\vec{v}| = 6$

11) Joel is walking on a tightrope over Niagara Falls. When he is 250 m from one side of the falls and 350 m from the other side, the tightrope is stretched downward a distance of 50 m. If Joel's mass is 48 kg, find the tension in each side of the rope. *(C – 4 marks)*





$\tan\alpha = \frac{50}{350}$

$\alpha \doteq 8.13°$

$\frac{|\vec{t}_1|}{\sin 78.69°} = \frac{470.4}{\sin 19.44°}$

$|\vec{t}_1| = 1385.9\text{ N}$

$\frac{|\vec{t}_2|}{\sin 81.87°} = \frac{470.4}{\sin 19.44°}$

$|\vec{t}_2| \doteq 1399.2\text{ N}$

$\tan\beta = \frac{50}{250}$

$\beta \doteq 11.31°$

∴ the tensions are 1385.9 N and 1399.2 N as shown in the diagram

12) Javid claims that it is impossible to have $\vec{u} \downarrow \vec{v} = \vec{v} \downarrow \vec{u}$. Is Javid's claim correct? Explain. *(C – 2 marks)*

Javid is incorrect. If $\vec{u}$ and $\vec{v}$ are perpendicular, then $\vec{u} \downarrow \vec{v} = \vec{v} \downarrow \vec{u}$. Specifically $\vec{u} \downarrow \vec{v}$ and $\vec{v} \downarrow \vec{u}$ are both $\vec{0}$.