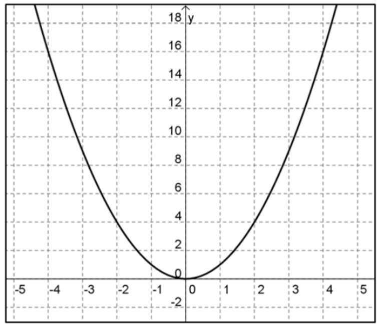
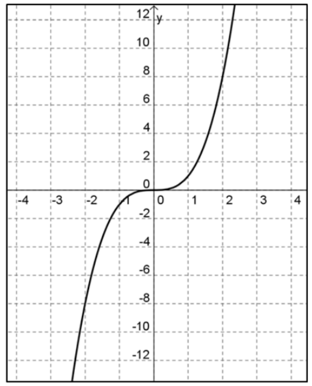
One-to-One Functions

Recall:

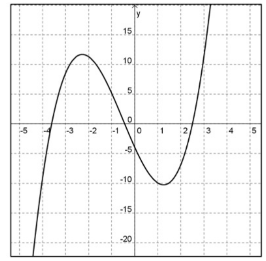
A **function** is a relation for which each element of the domain corresponds to exactly one element of the range. That is, for each *x*, there is exactly one *y*. These relations pass the vertical line test).

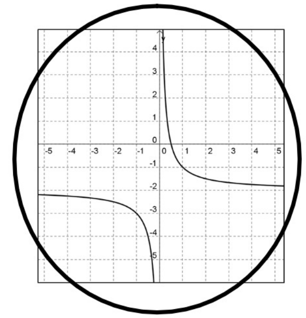


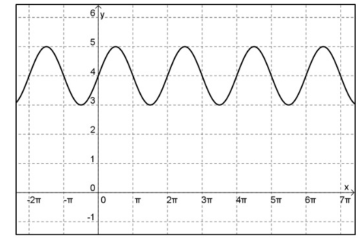
Furthermore, a function is **one-to-one** if we also have that each element of the range corresponds to exactly one element of the domain. That is, for each *y*, there is exactly one *x*. These relations pass the vertical line test and the horizontal line test.



Circle all of the **one-to-one functions**:







$y = |x+3|$

x	y
1	10
2	20
3	30
4	20
5	10

x	y
1	90
2	80
3	70
2	60
1	50

$f(x) = 4\sqrt{x+6} - 7$

$y = \dfrac{1}{x^2+5}$

$y = \pm\sqrt{x}$

$y = -2(5)^x + 7$

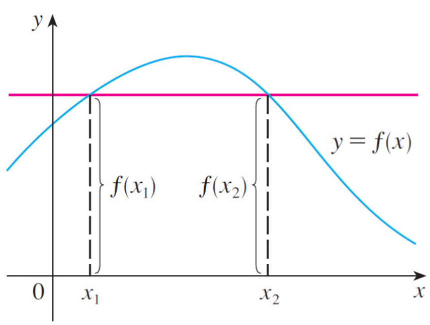
Getting Formal

DEFINITION One-to-One Function

A function $f(x)$ is **one-to-one** on a domain D if $f(a) \neq f(b)$ whenever $a \neq b$.

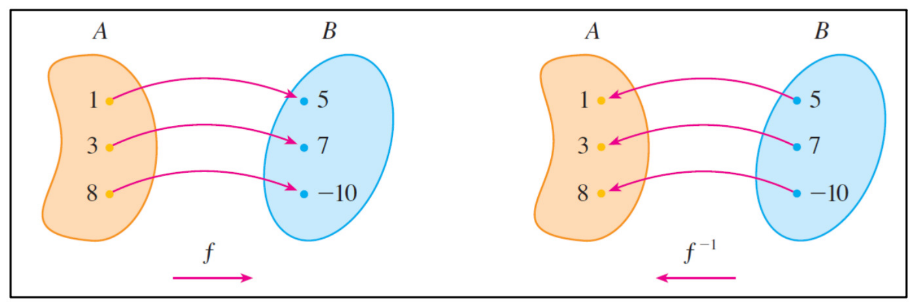
In other words, a function is one-to-one if it never takes on the same value twice.

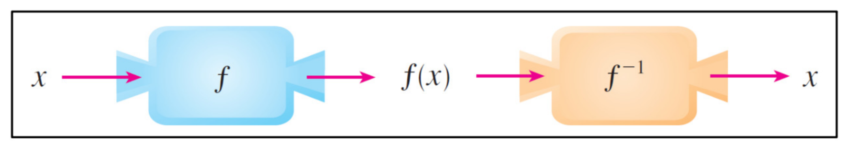
The function shown on the right is not a one-to-one function.



Inverses

Recall that the inverse of a function "undoes" or "reverses" the effect of the function.





$$f^{-1}(x) = y \iff f(y) = x$$

$$f^{-1}(f(x)) = x$$

$$f(f^{-1}(x)) = x$$

Determine the equation for the inverse of the following function.
$f(x)=2(x-5)^2-6$

$$y=2(x-5)^2-6$$

For inverse,

$$x=2(y-5)^2-6$$
$$x+6=2(y-5)^2$$
$$\frac{x+6}{2}=(y-5)^2$$
$$\pm\sqrt{\frac{x+6}{2}}=y-5$$
$$5\pm\sqrt{\frac{x+6}{2}}=y$$

∴ the inverse is

$$f^{-1}(x)=5\pm\sqrt{\frac{x+6}{2}}$$

Graphing the Inverse Parametrically

The use of parametric equations allows us to easily graph a function and its inverse using technology.

Graphing $y=f(x)$ and $y=f^{-1}(x)$ Parametrically

We can graph any function $y=f(x)$ as

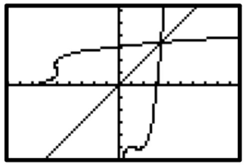
$$x_1=t,\quad y_1=f(t).$$

Interchanging t and $f(t)$ produces parametric equations for the inverse:

$$x_2=f(t),\quad y_2=t.$$

Consider the function $f(x)=x^3-4x^2+5x-7$.

1) Express $f(x)$ using parametric equations.
$$x=t$$
$$y=t^3-4t^2+5t-7$$
2) Express $f^{-1}(x)$ using parametric equations.
$$x=t^3-4t^2+5t-7$$
$$y=t$$



3) Use a calculator to graph $f(x)$, $f^{-1}(x)$ and the line $y=x$.

Logarithms

1) Evaluate the following logarithms.

a) $\log_2 32=5$ b) $\log_5\frac{1}{125}=-3$ c) $\log_4\sqrt[3]{16}=\frac{2}{3}$

2) Complete the following logarithm laws.

$$\log_a xy=\log_a x+\log_a y$$
$$\log_a\frac{x}{y}=\log_a x-\log_a y$$
$$\log_a x^y=y\log_a x$$
$$\log_a a^x=x \qquad a^{\log_a x}=x \qquad \log_a x=\frac{\log_b x}{\log_b a}$$

Special Logarithms

Common Logarithm	Natural Logarithm
$\log_{10}x=\log x$	$\log_e x=\ln x$

Examples

1) Solve the equation
$$2(1.75)^x=16$$
$$1.75^x=8$$
$$\log 1.75^x=\log 8$$
$$x\log 1.75=\log 8$$
$$x=\frac{\log 8}{\log 1.75}$$
$$x\doteq 3.72$$

2) Solve the equation
$$6(e)^{x+4}-81=130$$
$$6(e)^{x+4}=211$$
$$e^{x+4}=\frac{211}{6}$$
$$\ln e^{x+4}=\ln\frac{211}{6}$$
$$x+4=\ln\frac{211}{6}$$
$$x=\ln\frac{211}{6}-4$$
$$x\doteq -0.44$$

3) Solve the following equation for *y*.

$$\ln(y+8)-2x=3$$

$$\ln(y+8)=2x+3$$

$$y+8=e^{2x+3}$$

$$y=e^{2x+3}-8$$

4) Solve the following equation.

$$e^{x}-2e^{-x}=-1$$

$$\left(e^{x}\right)^{2}-2e^{0}=-e^{x}$$

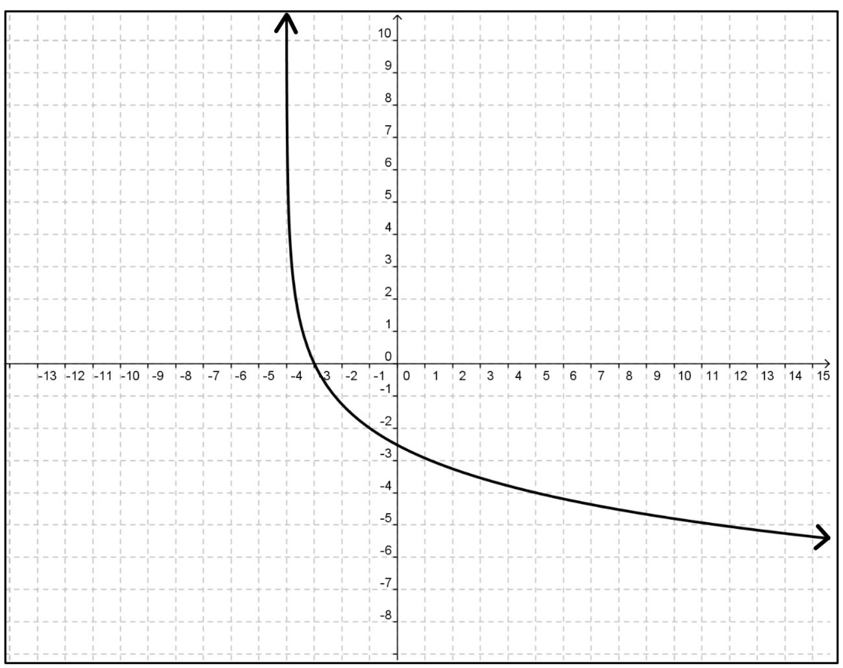
$$\left(e^{x}\right)^{2}+e^{x}-2=0$$

$$\left(e^{x}-1\right)\left(e^{x}+2\right)=0$$

$e^{x}-1=0$	$e^{x}+2=0$
$e^{x}=1$	$e^{x}=-2$
$\ln e^{x}=\ln 1$	$\ln e^{x}=\ln(-2)$
$x=0$	$x=\ln(-2)$
	No solution

$\therefore x=0$

5) Sketch the graph of the function $y=-2\log_{3}(x+4)$.



6) Evan invests \$1000 in an account that earns 5.25% compounded annually. How long will it take for the account to reach \$2500?

$$2500=1000(1.0525)^{n}$$

$$2.5=1.0525^{n}$$

$$\ln 2.5=\ln 1.0525^{n}$$

$$\ln 2.5=n\ln 1.0525$$

$$\frac{\ln 2.5}{\ln 1.0525}=n$$

$$n\doteq 17.9$$

$\therefore$ it will take approximately 17.9 years.