FACTORING POLYNOMIALS – REFINING THE PROCEDURE

What have we learned?

For a polynomial $P(x)$, if we can find a number k such that $P(k)=0$, then…..

… we will know that the remainder when we divide $P(x)$ by $x-k$ will be zero, so …

… $x-k$ is a factor of $P(x)$.

Formally,

The Remainder Theorem

When a polynomial, $P(x)$, is divided by $x – k$, the remainder is equal to $P(k)$.

When a polynomial, $P(x)$, is divided by $jx – k$, the remainder is equal to $P\left(\frac{k}{j}\right)$.

The Factor Theorem

$x – k$ is a factor of $P(x)$ if and only if $P(k) = 0$.

$jx – k$ is a factor of $P(x)$ if and only if $P\left(\frac{k}{j}\right)=0$.

So, here's the procedure! To factor a polynomial, $P(x)$:

1) Find a k value such that $P(k)=0$.
2) Now you know that $(x-k)$ is a factor of $P(x)$, so divide $P(x)$ by $(x-k)$ to find the other factor. (The remainder *must* work out to zero)
3) Repeat the process until you get all factors down to degree 2.
4) Factor any degree 2 factors, using methods from grades 10 and 11.

One more helpful fact…

In any polynomial that can be factored, the linear factors have the form $x – k$ or $jx – k$. As a result, any zero of the polynomial will be of the form $\frac{p}{q}$, where p is a factor of the constant term and q is a factor of the leading coefficient.

Example: Factor $2x^4 - 9x^3 - x^2 + 18x + 8$.

Example: Determine if $2x - 5$, is a factor of the polynomial $2x^3 - 5x^2 - 2x + 5$.

Example: Find k such that $x - 3$ is a factor of $f(x) = 2x^3 - x^2 + kx + 36$.