EQUATION OF A LINE IN 2-SPACE

Equation of a Line in 2-Space

What information do we need in order to determine a straight line?

1) _____

or 2) _____

Instead of specifying the direction of a line using its slope, we could use a vector. A vector used to describe the direction of a straight line is called a _____.

Direction Vector

A **direction vector** is any non-zero vector that is parallel to the line being described.

- Notice that a single line has infinitely many different direction vectors.

Example

Find a direction vector for each line.

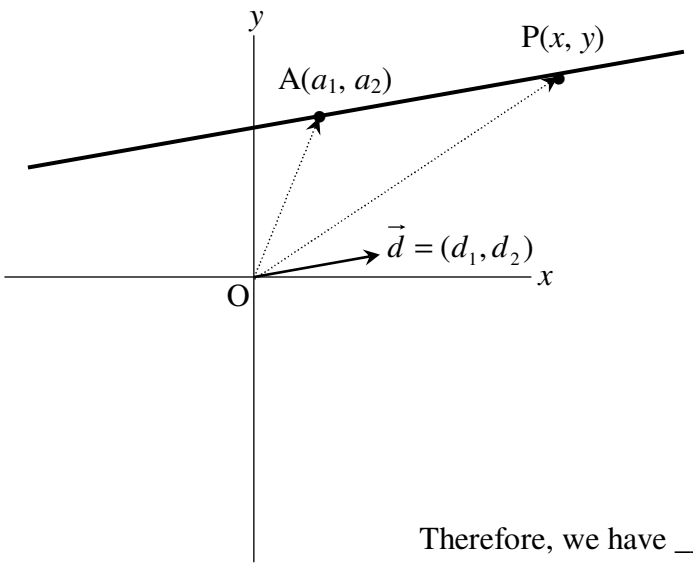
a) The line passing through the points $A(4, -5)$ and $B(3, -7)$.

b) A line with slope $\frac{4}{5}$.

Using a direction vector to specify the "slant" of a line and one point on the line to fix its location on the plane, we can develop a **vector equation** of the line.

Vector Equation of a Line in 2-Space

Consider the following diagram, in which A(a_1, a_2) is a known point on the given line and P(x, y) is any point on the line. Also, let $\vec{d} = (d_1, d_2)$ be a direction vector of the given line.



Notice that regardless of the location of point P on the line, we always have $\overrightarrow{OP} = \overrightarrow{OA} + \overrightarrow{AP}$.

Furthermore, by definition, $\vec{d}$ is parallel to $\overrightarrow{AP}$.

Therefore, $\overrightarrow{AP} =$

Also, $\overrightarrow{OP} =$ and $\overrightarrow{OA} =$

Therefore, we have _____.

The above equation is called a **vector equation** of the line.

- Can a line have more than one vector equation?

Vector Equation of a Line in 2-Space

Let A(a_1, a_2) be a fixed point on a line in 2-space with a direction vector $\vec{d} = (d_1, d_2)$ and let P(x, y) be any point on the line. A **vector equation** of the line is

$$(x, y) = (a_1, a_2) + t(d_1, d_2)$$

where t is any scalar.

By choosing different values for t in a vector equation, we can determine various points on the line.

Example
A line has a direction vector (4, -2) and passes through the point (3, 5). Find the vector equation of the line and 3 points on the line.

Parametric Equations of a Line in 2-Space

In the vector equation $(x, y) = (a_1, a_2) + t(d_1, d_2)$, the scalar t is called a parameter. By simplifying the right-hand side of the vector equation and equating the corresponding components of the equal vectors, we can develop equations for x and y in terms of t.

The two equations above are called **parametric equations** of the line.

- Are the parametric equations of a line unique?

Parametric Equations of a Line in 2-Space

Let $A(a_1, a_2)$ be a fixed point on a line in 2-space with a direction vector $\vec{d} = (d_1, d_2)$ and let $P(x, y)$ be any point on the line. **Parametric equations** of the line are

$$x = a_1 + td_1$$
$$y = a_2 + td_2$$

where t is any scalar.

Example
Find a vector equation and parametric equations of the line through the points (4, 7) and (-8, 2) and determine where this line crosses the *x*-axis.

Example
Determine a vector equation for the line that is perpendicular to $\vec{r} = (4,1) + t(-3,2)$ and that passes through the point (6, 5).