SOLUTIONS

MHF4U1 - UNIT 7 – TRIGONOMETRIC IDENTITIES AND EQUATIONS
SUPPLEMENTARY REVIEW PROBLEMS

1) Express each of the following as a **single** trigonometric function. Simplify as much as possible.

a) $5\sin\frac{\theta}{3}\cos\frac{\theta}{3}$

$= 2.5\left(2\sin\frac{\theta}{3}\cos\frac{\theta}{3}\right)$

$= 2.5\sin\left(2\times\frac{\theta}{3}\right)$

$= 2.5\sin\frac{2\theta}{3}$

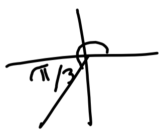
b) $\dfrac{1-\tan^2\frac{\theta}{2}}{2\tan\frac{\theta}{2}}$

$= \cot\left(2\times\frac{\theta}{2}\right)$

$= \cot\theta$

2) Determine the exact value of each of the following. You do not need to rationalize the denominators in your answers.

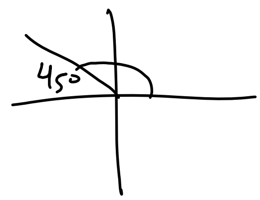
a) $\cos\frac{4\pi}{3}$



$= -\cos\frac{\pi}{3}$

$= -\frac{1}{2}$

b) $\csc 135°$



$= \csc 45°$

$= \sqrt{2}$

c) $\cos\frac{7\pi}{12}$

$= \cos\left(\frac{3\pi}{12}+\frac{4\pi}{12}\right)$

$= \cos\left(\frac{\pi}{4}+\frac{\pi}{3}\right)$

$= \cos\frac{\pi}{4}\cos\frac{\pi}{3} - \sin\frac{\pi}{4}\sin\frac{\pi}{3}$

$= \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right) - \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right)$

$= \frac{1}{2\sqrt{2}} - \frac{\sqrt{3}}{2\sqrt{2}}$

$= \frac{1-\sqrt{3}}{2\sqrt{2}}$

d) $\left(\sin\frac{\pi}{8}-\cos\frac{\pi}{8}\right)^2$

$= \left(\sin\frac{\pi}{8}-\cos\frac{\pi}{8}\right)\left(\sin\frac{\pi}{8}-\cos\frac{\pi}{8}\right)$

$= \sin^2\frac{\pi}{8} - \sin\frac{\pi}{8}\cos\frac{\pi}{8} - \cos\frac{\pi}{8}\sin\frac{\pi}{8} + \cos^2\frac{\pi}{8}$

$= \sin^2\frac{\pi}{8} + \cos^2\frac{\pi}{8} - 2\sin\frac{\pi}{8}\cos\frac{\pi}{8}$

$= 1 - 2\sin\frac{\pi}{8}\cos\frac{\pi}{8}$

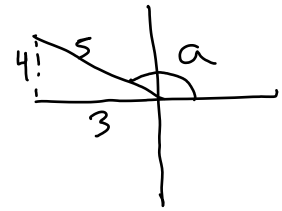
$= 1 - \sin\left(2\times\frac{\pi}{8}\right)$

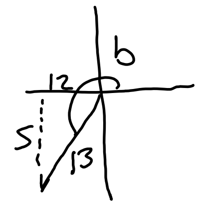
$= 1 - \sin\frac{\pi}{4}$

$= 1 - \frac{1}{\sqrt{2}}$

$= \frac{\sqrt{2}-1}{\sqrt{2}}$

3) Let $\frac{\pi}{2} \le a \le \pi$ and $\pi \le b \le \frac{3\pi}{2}$. If $\sin a = \frac{4}{5}$ and $\cos b = -\frac{12}{13}$, find the exact value of $\cos(a-b)$.





$$\cos(a-b) = \cos a \cos b + \sin a \sin b$$
$$= \left(-\frac{3}{5}\right)\left(-\frac{12}{13}\right) + \left(\frac{4}{5}\right)\left(-\frac{5}{13}\right)$$
$$= \frac{36}{65} - \frac{20}{65}$$
$$= \frac{16}{65}$$

4) Prove the following identities.

a) $\frac{\sin x \cos x}{\tan x} = 1 - \sin^2 x$

L.S.

$$\frac{\sin x \cos x}{\tan x}$$
$$= \frac{\sin x \cos x}{\frac{\sin x}{\cos x}}$$
$$= \cancel{\sin x} \cos x \times \frac{\cos x}{\cancel{\sin x}}$$
$$= \cos^2 x$$

R.S.

$$1 - \sin^2 x$$
$$= \cos^2 x$$

$\therefore$ L.S. = R.S.

$\therefore$ identity proven

b) $\frac{\sin 2x}{\sin x} - \frac{\cos 2x}{\cos x} = \sec x$

L.S.

$$\frac{\sin 2x}{\sin x} - \frac{\cos 2x}{\cos x}$$
$$= \frac{2\cancel{\sin x}\cos x}{\cancel{\sin x}} - \frac{\cos 2x}{\cos x}$$
$$= 2\cos x - \frac{\cos 2x}{\cos x}$$
$$= \frac{2\cos^2 x - \cos 2x}{\cos x}$$
$$= \frac{2\cos^2 x - (2\cos^2 x - 1)}{\cos x}$$
$$= \frac{2\cos^2 x - 2\cos^2 x + 1}{\cos x}$$
$$= \frac{1}{\cos x}$$

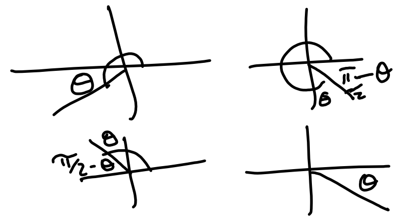
R.S.

$$\sec x$$
$$= \frac{1}{\cos x}$$

$\therefore$ L.S. = R.S.

$\therefore$ identity proven

c) $\dfrac{\sin(\pi+\theta)\tan\left(\dfrac{3\pi}{2}+\theta\right)}{\cos\left(\dfrac{\pi}{2}+\theta\right)\csc(-\theta)} = \cos\theta$



L.S.

$$\frac{\sin(\pi+\theta)\tan\left(\frac{3\pi}{2}+\theta\right)}{\cos\left(\frac{\pi}{2}+\theta\right)\csc(-\theta)}$$

$$= \frac{-\sin\theta\left[-\tan\left(\frac{\pi}{2}-\theta\right)\right]}{-\cos\left(\frac{\pi}{2}-\theta\right)\left(-\csc\theta\right)}$$

$$= \frac{-\cancel{\sin\theta}\,(-\cot\theta)}{-\cancel{\sin\theta}\,(-\csc\theta)}$$

$$= \frac{\cot\theta}{\csc\theta}$$

$$= \frac{\frac{\cos\theta}{\sin\theta}}{\frac{1}{\sin\theta}}$$

$$= \frac{\cos\theta}{\cancel{\sin\theta}} \times \frac{\cancel{\sin\theta}}{1}$$

$$= \cos\theta$$

R.S.

$\cos\theta$

$\therefore$ L.S. = R.S.

$\therefore$ identity proven

5) Solve the following equations for $0° \le x \le 360°$. Round your answer to the nearest tenth of a degree.

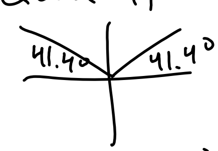
a) $\csc x = 1.5124$

$\sin x = \dfrac{1}{1.5124}$

R.A.A. = 41.4°

sin is +ve

Quad 1,2



$\therefore x = 41.4°$ or $138.6°$

b) $\sin 3x = -0.5$

$0° \le x \le 360°$

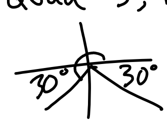
$0° \le 3x \le 1080°$

$\sin 3x = -\dfrac{1}{2}$

R.A.A. = 30°

Sin is −ve

Quad 3,4



$\therefore 3x = 210°, 570°, 930°, 330°, 690°, 1050°$

$x = 70°, 110°, 190°, 230°, 310°, 350°$

c) $\cos 2x = \sin x$

$\cos 2x - \sin x = 0$

$1 - 2\sin^2 x - \sin x = 0$

$0 = 2\sin^2 x + \sin x - 1$

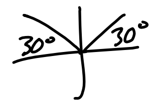
$0 = (2\sin x - 1)(\sin x + 1)$

$\sin x = \frac{1}{2}$ or $\sin x = -1$

R.A.A. $= 30°$

sin is +ve

Quad 1, 2



$\therefore x = 30°, 150°$

$x = 270°$

$\therefore x = 30°, 150°, 270°$

6) Solve the following equations for $0 \le \theta \le 2\pi$. Give exact solutions, where possible. Round approximate solutions to the nearest thousandth of a radian.

a) $\sin^2 \theta - 2\sin\theta - 3 = 0$

$(\sin\theta - 3)(\sin\theta + 1) = 0$

$\sin\theta = 3$

No solution

$\sin\theta = -1$

$\theta = \frac{3\pi}{2}$

b) $\sin\theta - 3\sin\theta\cos\theta = 0$

$\sin\theta(1 - 3\cos\theta) = 0$

$\sin\theta = 0$

$\theta = 0, \pi, 2\pi$

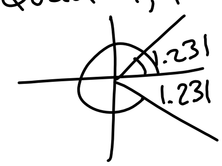
$1 - 3\cos\theta = 0$

$\cos\theta = \frac{1}{3}$

R.A.A. $\doteq 1.231$

cos is +ve

Quad 1, 4



$\theta = 1.231$ or

$\theta = 2\pi - 1.231$

$\doteq 5.052$

$\therefore \theta = 0, 1.231, \pi, 5.052, 2\pi$

7) Using the addition formula for cosine, derive the double angle formulas for cosine.

$\cos 2x = \cos(x + x)$

$= \cos x \cos x - \sin x \sin x$

(1) $= \cos^2 x - \sin^2 x$

$= (1 - \sin^2 x) - \sin^2 x$

(2) $= 1 - 2\sin^2 x$

$= \cos^2 x - (1 - \cos^2 x)$

$= \cos^2 x - 1 + \cos^2 x$

(3) $= 2\cos^2 x - 1$